Basics of Unsaturated Soil Mechanics

SLOPE STABILITY COURSE

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EPFL / ENAC / GC section – Master semester 2 and 4 – 2024-2025

EPFL

Outline

- Basic concepts
  - Introduction
  - The definition of suction

- Hydraulic properties of soils under unsaturated conditions
  - The water retention curve and its hysteretic feature
  - The permeability

- Mechanical behavior of unsaturated soils
  - Net stress and generalized effective stress
  - Possible stress paths
  - Volumetric behavior
  - Shear strength

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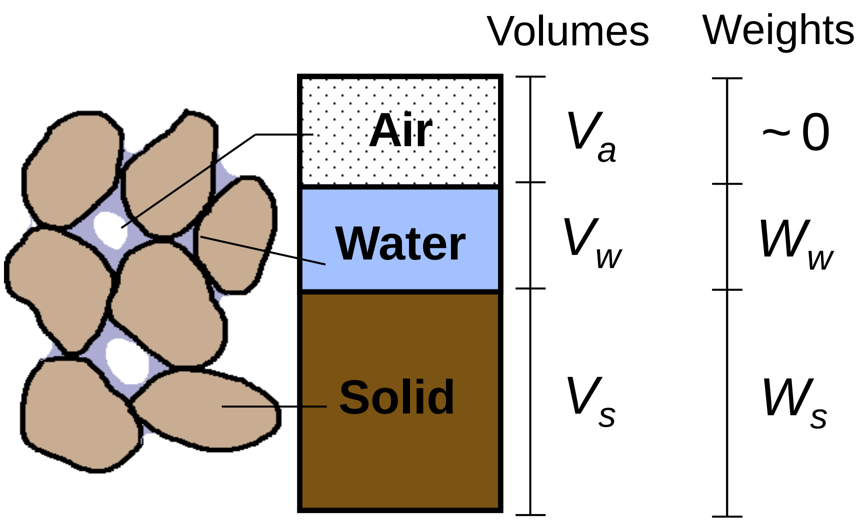


Three-phase description

- Volume of voids $V_v = V_a + V_w$
- Total volume $V = V_v + V_s$
- Porosity $n = V_v/V$
- Void ratio $e = V_v/V_s$

The amount of water in the soil can be quantified in different ways:

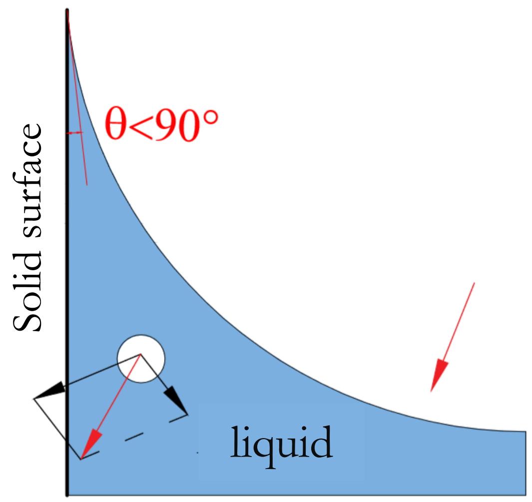
- Gravimetric Water content $w = W_w/W_s$
- Degree of saturation $S_r = V_w/V_v$
- Volumetric water content $\theta = V_w/V$
- Water ratio $e_w = V_w/V_s$

Unsaturated soils are soils whose voids are occupied by water and air.

Capillary systems

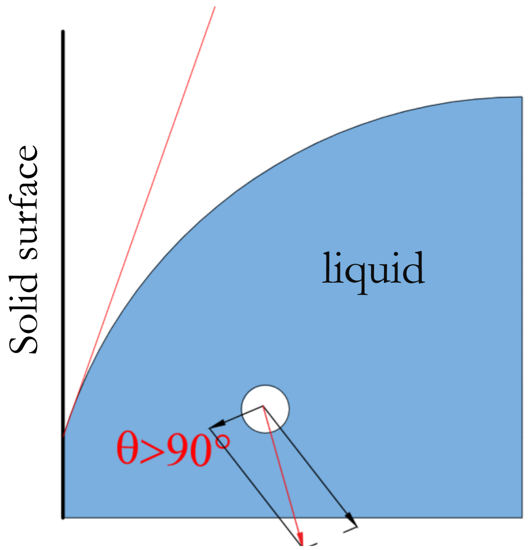
- **Adhesion** is the attraction force between molecules of different type.
- **Cohesion** is the attraction force between molecules of the same type.

Adhesion> Cohesion



The liquid "wets" the surface

Adhesion < Cohesion



The liquid does not wet the surface

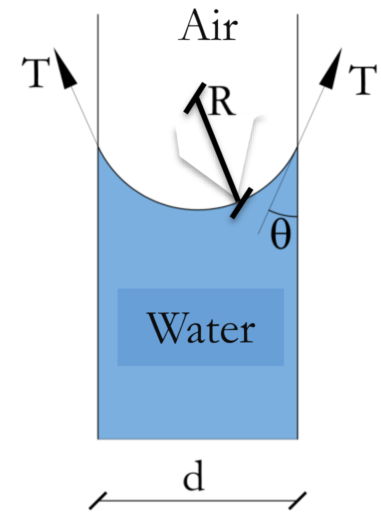
- When in contact with a solid surface, the interface will curve near that surface to form a **meniscus**
- The contact angle θ, which is measured through the liquid, lower than 90° is typical for soil water on soil minerals

Capillary systems

- Equilibrium of the interphase in the vertical direction :

$$u_a \pi \frac{d^2}{4} = u_w \pi \frac{d^2}{4} + 2\pi \frac{d}{2} T \cos \vartheta$$

$$u_a - u_w = \frac{4T\cos(\theta)}{d} = \frac{2T}{R}$$



u_a = air pressure [F/L^2]

u_w = water pressure [F/L^2]

θ = contact angle

T = surface tension [F/L]

d = diameter of capillary tube [L]

R = radius of curvature of spherical cup [L]

Suction

The difference between pore air pressure and pore water pressure is named matric suction

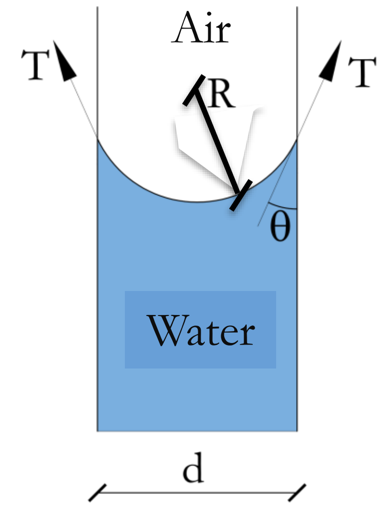
$$s = u_a - u_w$$

In a capillary tube, suction depends on the size of the pore (d) and the curvature of the interface (θ)

$$s = u_a - u_w = \frac{4T\cos(\theta)}{d}$$

Values of suction we can expect in soils

$$s = u_a - u_w = \frac{4T\cos(\theta)}{d}$$



$$\vartheta = 0$$

$$T = 0.073 N/m \; (20^{\circ}\ \mathrm{C})$$

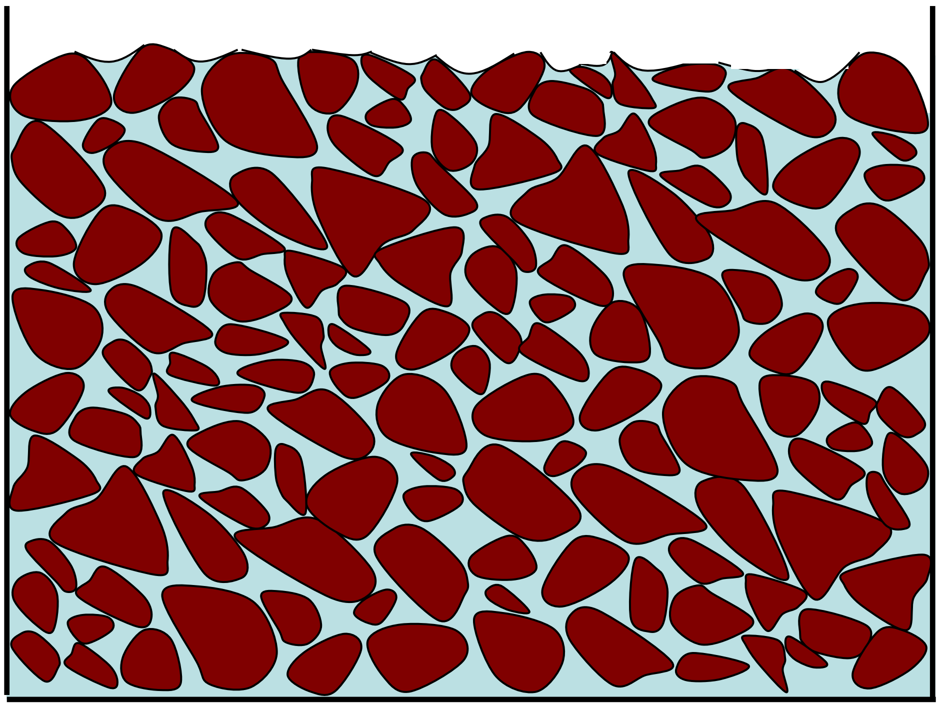
	Sand	Silt	Clay
d grain (mm)	2	0.075	0.002
d pore (mm)	0.2	0.0075	0.0002
u_a - u_w (kPa)	1.4	38	1440

Relative pore water pressure are negative

Pore water pressure can be negative in absolute values (!)

The water retention curve

SATURATED STATE

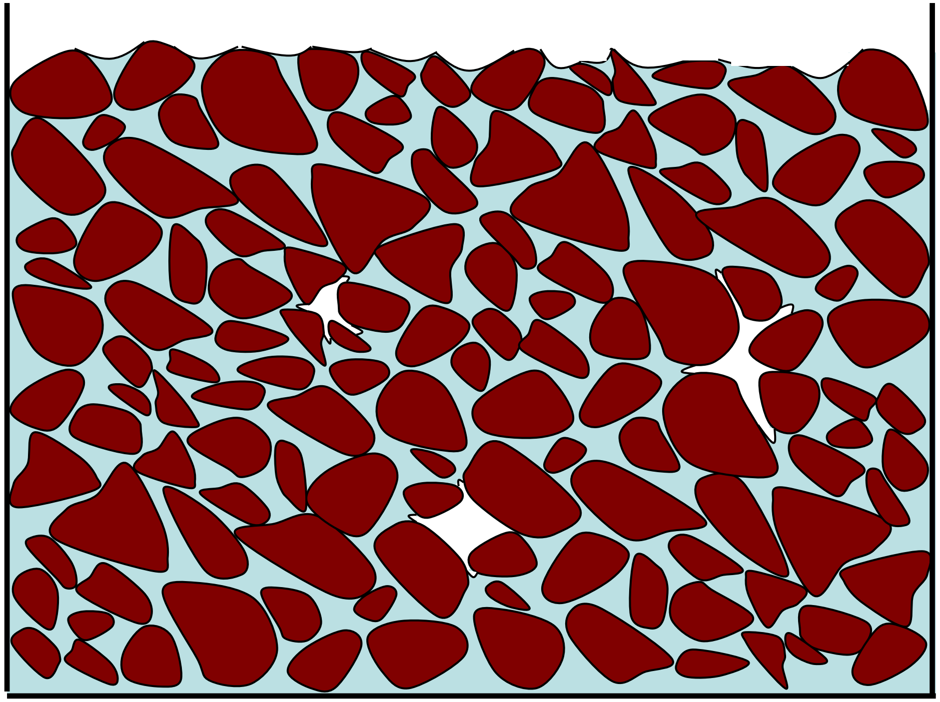


An initially saturated soil is subjected to evaporation

- **Menisci** at the surface start to appear
- The pore water pressure decreases and the suction becomes higher than 0, **s>0**
- The soil is still saturated, **S_r=1**

The water retention curve

QUASI - SATURATED STATE

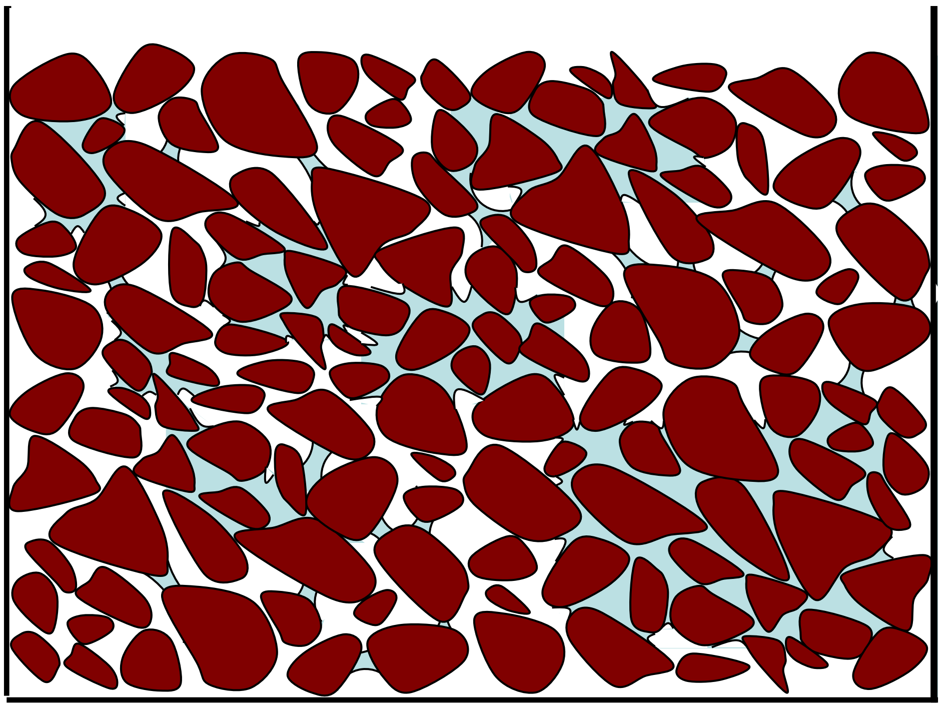


Indicative values of the degree of saturation $0.85\text{-}0.90 < S_r < 1$

- More menisci appear, air cavities in the larger pores expand
- The pore water pressure continues decreasing and the suction continue increasing, **$s>0$**
- **Gas** phase is **discontinuous**, **liquid** phase is **continuous**
- The degree of saturation decreases**,** **$S_r<1$**

The water retention curve

PARTIALLY SATURATED STATE

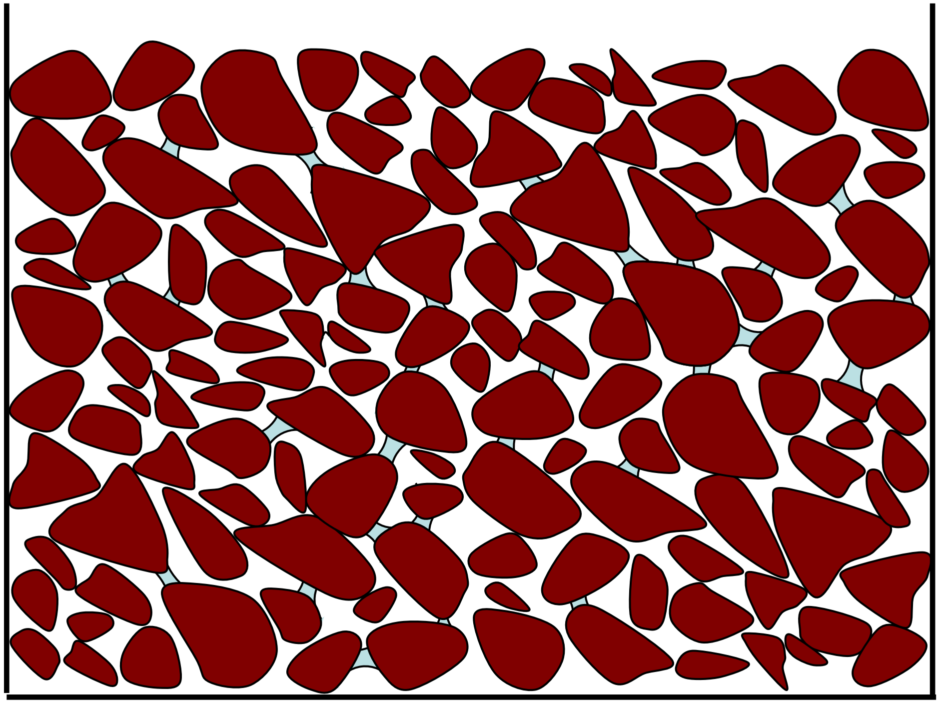


Indicative values of the degree of saturation
$0.1 < S_r < 0.85\text{-}0.90$

- More menisci appear, also the smaller pores start to desaturate; at the interface the maximum curvature of the menisci is reached and the air enter
- The pore water pressure continues decreasing and the suction continues increasing, **$s>0$**
- **Gas** phase is **continuous**, **liquid** phase is **continuous**
- The degree of saturation decreases, **$S_r<1$**

The water retention curve

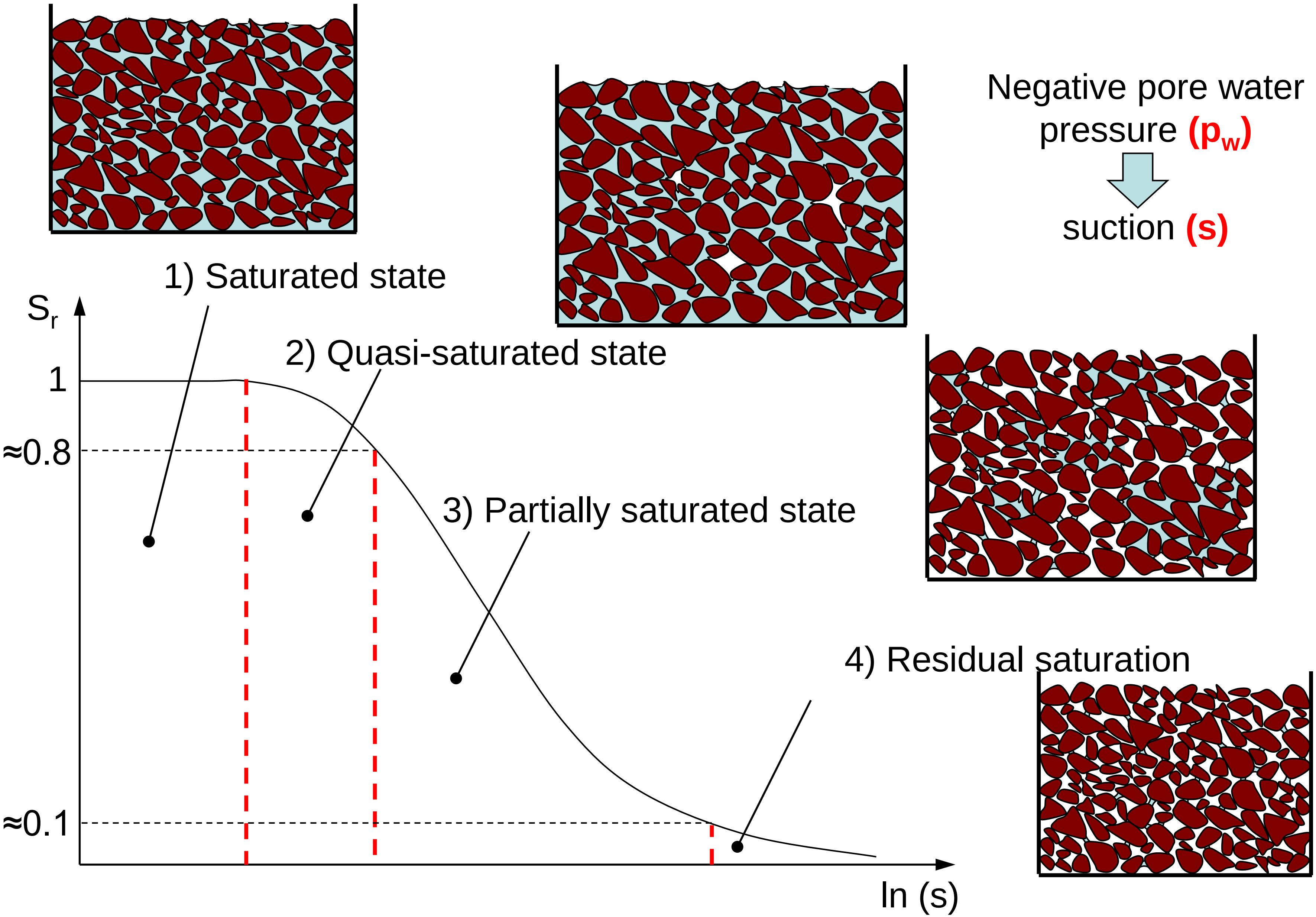
RESIDUAL STATE



Indicative values of the degree of saturation
$S_r < 0.1$

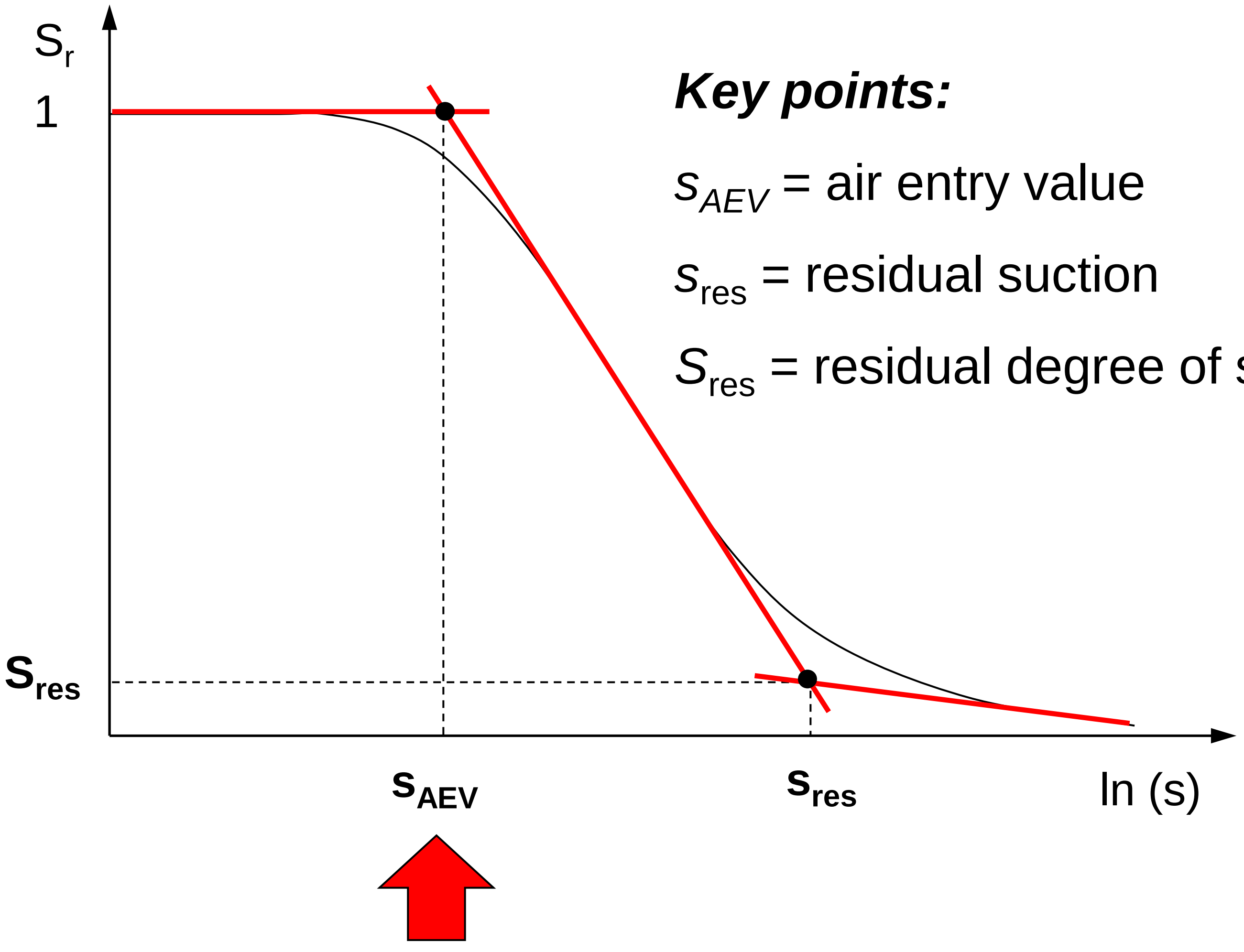
- There is water in some isolated pores
- The pore water pressure continues decreasing and the suction continues increasing, **s>0**
- **Gas** phase is **continuous**, **liquid** phase is **discontinuous**
- The degree of saturation does not decrease significantly, **$S_r<1$**

The water retention curve



The water retention curve

The relationship between the degree of saturation and the suction is called “water retention curve”. It expresses the capacity of the soil to retain water.



Key points:

s_{AEV} = air entry value

s_{res} = residual suction

S_{res} = residual degree of saturation

The water retention curve

THE AIR ENTRY VALUE

- The air entry value (s_{AEV}) is the approximate value of suction starting from which the soil starts to desaturate.
- The order of magnitude of s_{AEV} is a function of the pores size: the larger the pores the lower is the suction needed for desaturating the specimen.
- s_{AEV} can be computed by using the Young-Laplace equation

Young-Laplace equation

$$s = u_a - u_w = \frac{4T\cos\theta}{d}$$

D: pore diameter for different geomaterials

Values assumed for the other parameters
T = 0.072 N/m
θ = 0°

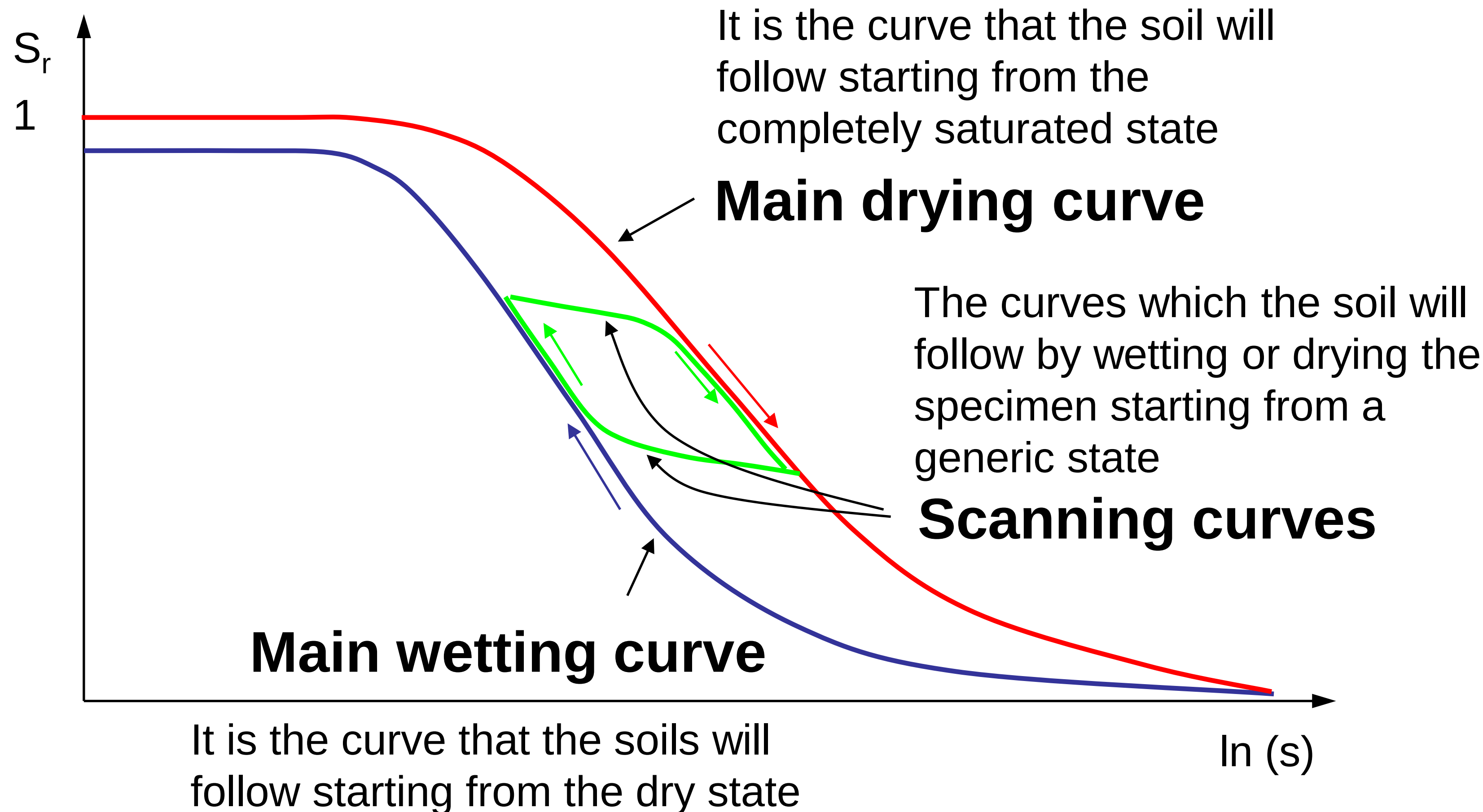
Order of magnitude of the air entry value depending on the characteristic pore size:

sand (d_{max}=30 μm)	**silt (d_{max}=3 μm)**	**clay (d_{max}=0.3 μm)**
9.6 kPa	**96 kPa**	**960 kPa**

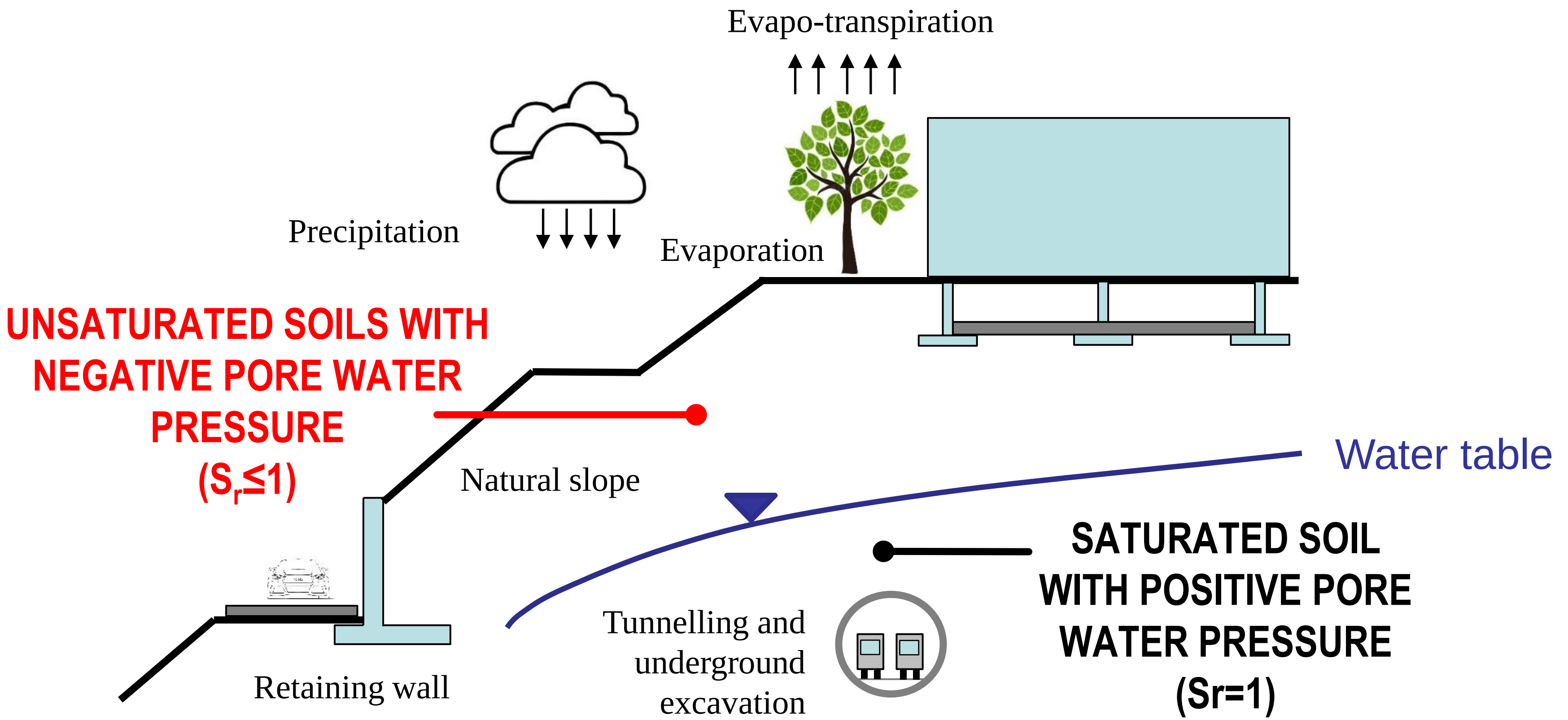
The soil water retention curve

DRYING AND WETTING PROCESSES – AN HYSTERETIC BEHAVIOR

- A process characterized by an increase in suction is called “drying process”
- A process characterized by a decrease in suction is called “wetting process”
- The relationship between suction and degree of saturation changes depending on the alternation of these processes



Above the water table the pore water pressure is negative and the soil can be unsaturated. Rainfall and evaporation phenomena are example of processes affecting pore water pressure and degree of saturation.

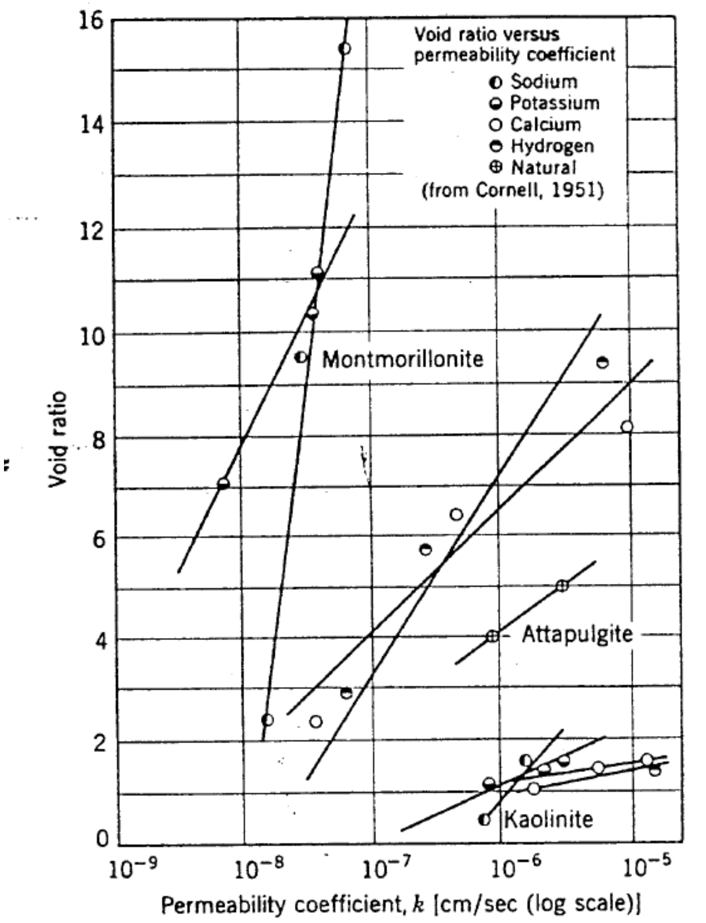


Permeability

The **coefficient of permeability** is the coefficient of proportionality between the flow rate and the hydraulic head gradient.

$$v_{w,x} = -k_w \frac{\partial h_w}{\partial x}$$

Darcy's law (example for the x-direction)



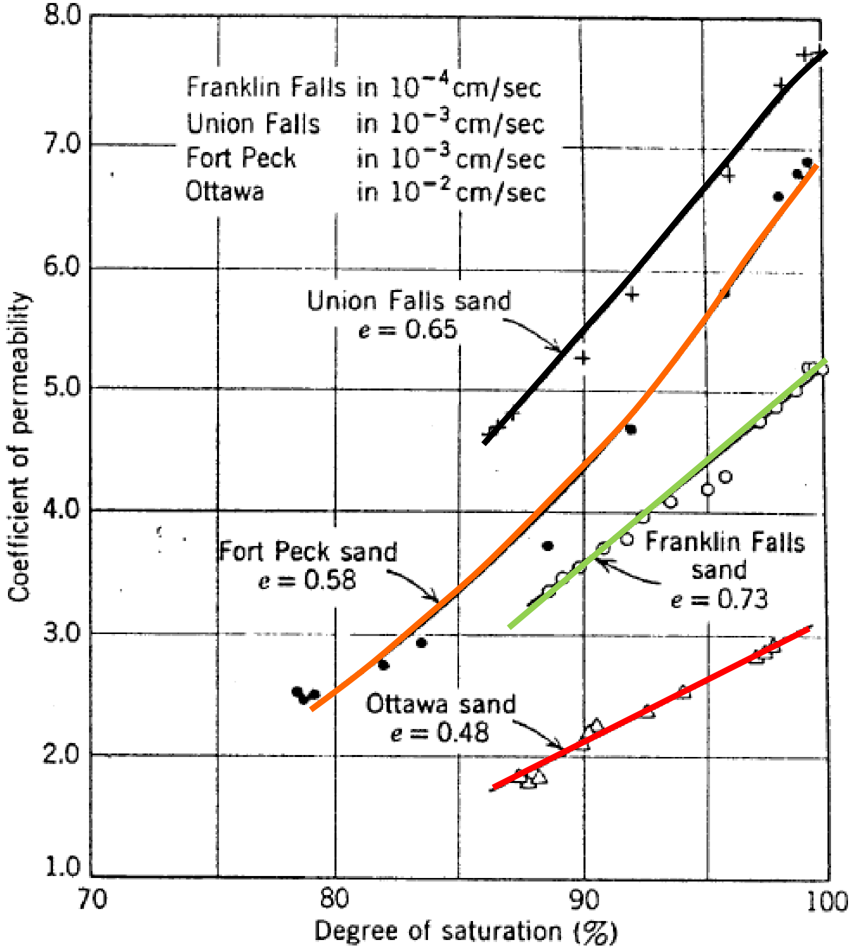
Example of test results
from Lambe and Whitman, 1969

For a given **saturated soil**, the coefficient of permeability mainly depends on the void ratio.

The higher the void ratio, the higher is the permeability coefficient.

Permeability

For an **unsaturated soil**, the Darcy's law is still valid but the coefficient of permeability depends on both the water content (or degree of saturation) and the void ratio.

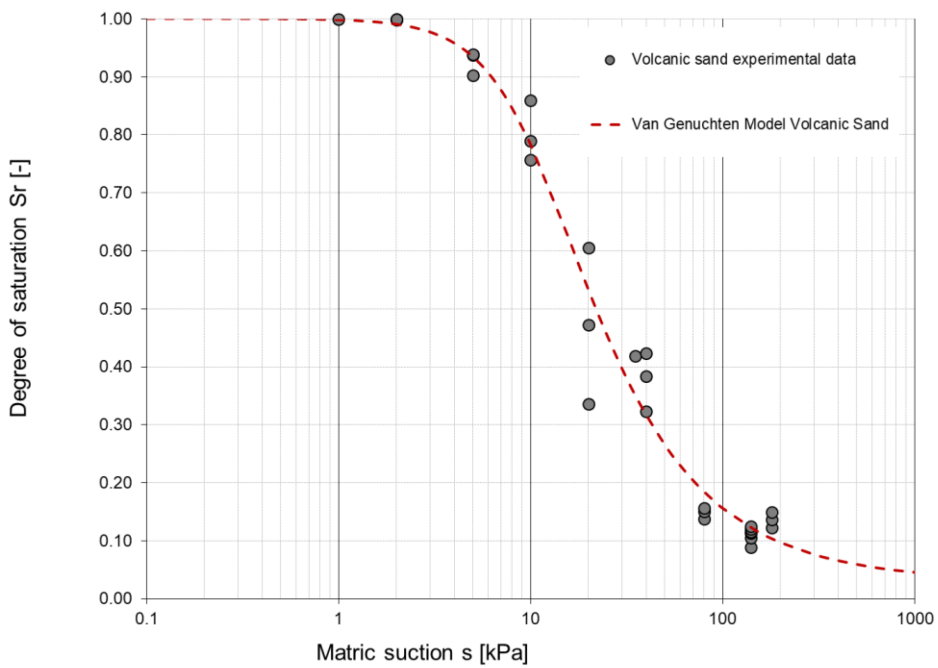


The more water is available in the soil, the more there are connected pores filled with water that the water can use for flowing.

For a given void ratio, the higher the degree of saturation the higher is the permeability coefficient.

Example of test results
from Lambe and Whitman, 1969

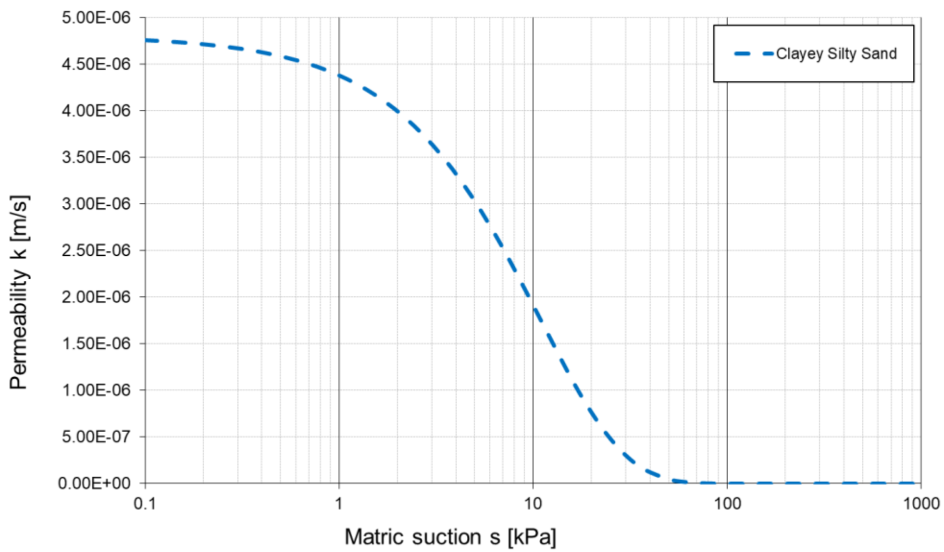
Water retention and permeability



Van Genuchten, M., 1980

$$S_r = \left\{\frac{1}{1+[\alpha(p_a - p_w)\,]^n}\right\}^m$$

α ,n, m calibration parameters

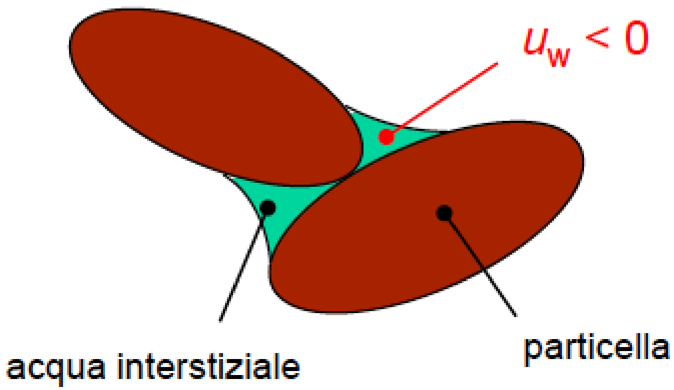


The evolution of the coefficient of permeability with suction can be described for example by using the Gardner's model (1958).

Gardner's model

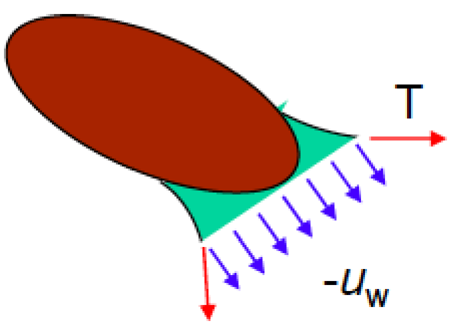
$$k = k_s e^{-\alpha(p_a - p_w)}$$

α fitting parameter



The contact angle of water with the particle surface is less than 90°

The meniscus is concave toward the air side and pore water presure is negative



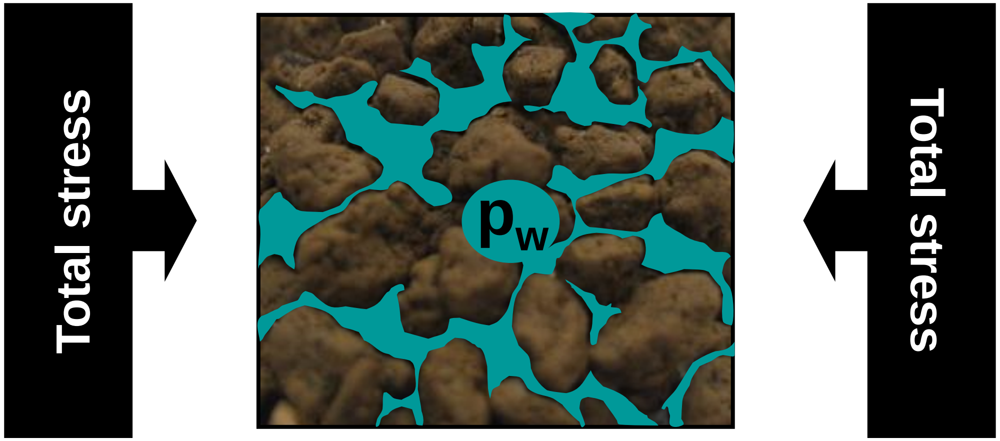
Particles are stuck together by surface tension and negative pressure

The Daytona Beach race (Florida, United States)

The sand castle and the possibility of having races in the beach are example of how the negative pore water pressure affect the mechanical properties of soils.

Suction, net stress and eff. stress

In the case of saturated specimens with incompressible water and solid grains the effective stress is



Terzaghi's effective stress (1936):

$$\sigma'_{ij} = \sigma_{ij} - p_w \delta_{ij}$$

Total stress

Pore water pressure

Suction, net stress and eff. stress

The difference between the pressure of the air and the pressure of the water is called **suction**

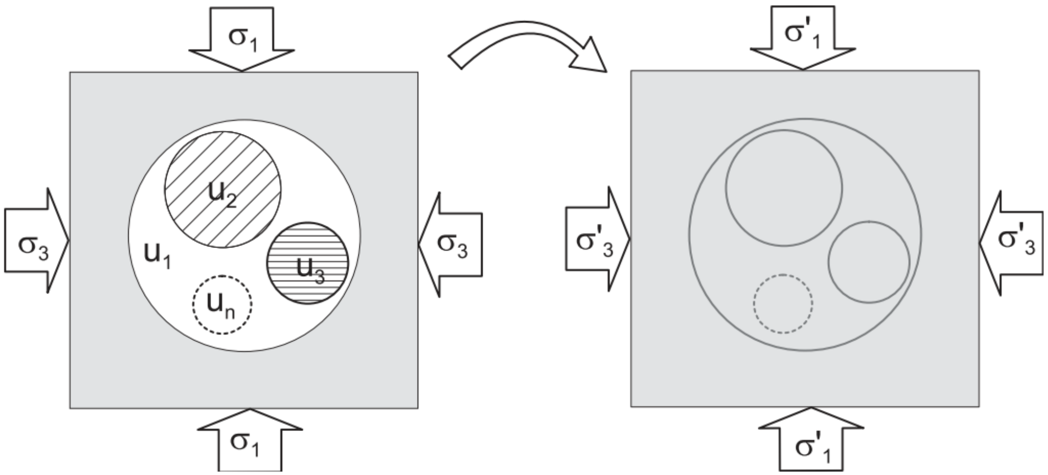
$$s = p_a - p_w$$

The difference between the external stress and the pressure of the air is called **net stress**

$\sigma_{net,ij} = \sigma_{ij} - p_a \delta_{ij}$ net stress tensor

Suction, net stress and eff. stress

The effective stress in presence of different fluids



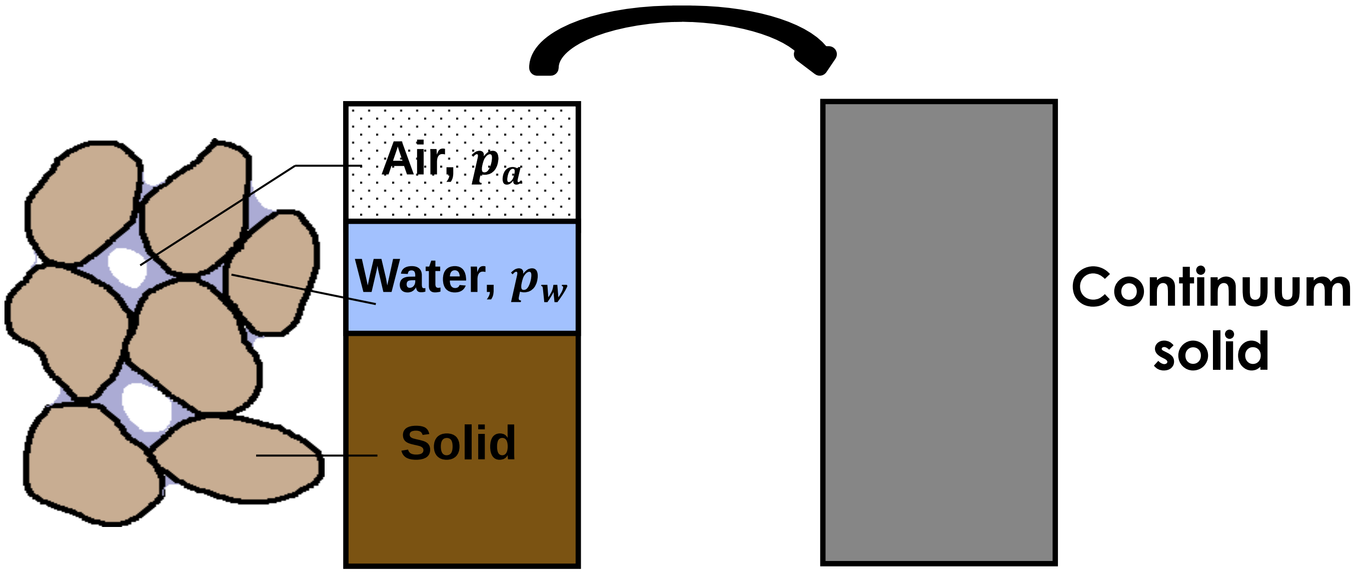
Conversion of a multiphase and multi stress medium in an equivalent medium

$$\sigma'_{ij} = \sigma_{ij} - \sum_{\beta=1}^{i} \alpha_\beta p_\beta \delta_{ij}$$

σ_{ij} : total stress
β: generic fluid
α_β: scaling factor of the fluid β
p_β: pressure of the fluid β

Suction, net stress and eff. stress

The effective stress in the specific case of unsaturated soils



Multi-phase description

Single-phase description

$$\sigma'_{ij} = \sigma_{ij} - \sum_{\beta=1}^{2} \alpha_\beta p_\beta \delta_{ij}$$

$$\sigma'_{ij} = \sigma_{ij} - (1-\chi) p_a \delta_{ij} - \chi p_w \delta_{ij}$$

$$\sigma'_{ij} = (\sigma_{ij} - p_a \delta_{ij}) + \chi (p_a - p_w)\, \delta_{ij}$$

$$\sigma'_{ij} = \sigma_{net,ij} + \chi s \delta_{ij}$$

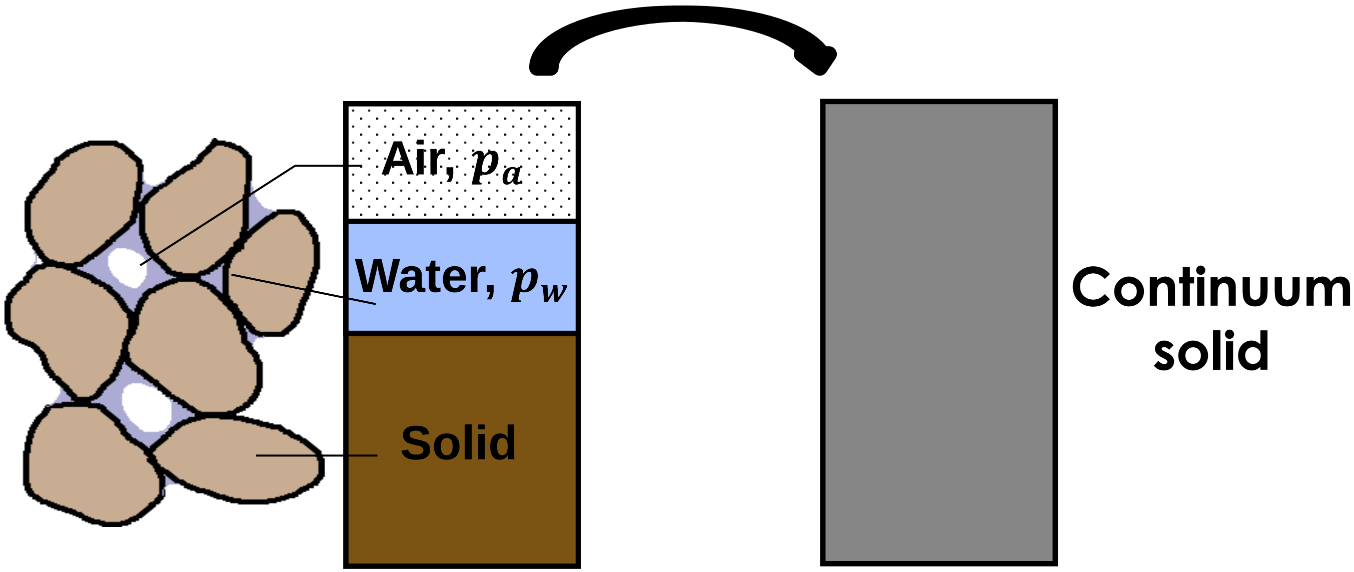
Net stress tensor

Suction stress tensor

with $\chi = f(S_r)$

Suction, net stress and eff. stress

The effective stress in the specific case of unsaturated soils



Multi-phase description

Single-phase description

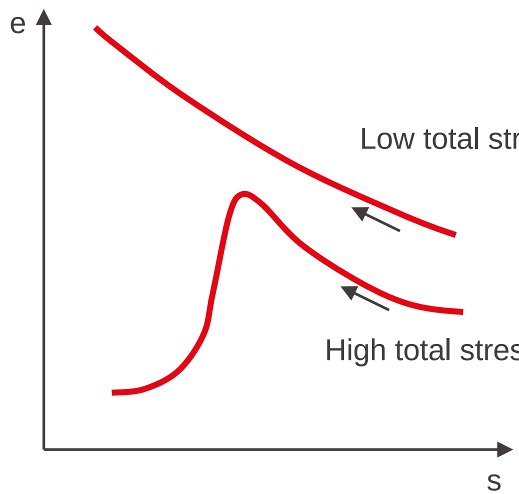
$$\sigma'_{ij} = \sigma_{net,ij} + \chi s \delta_{ij}$$

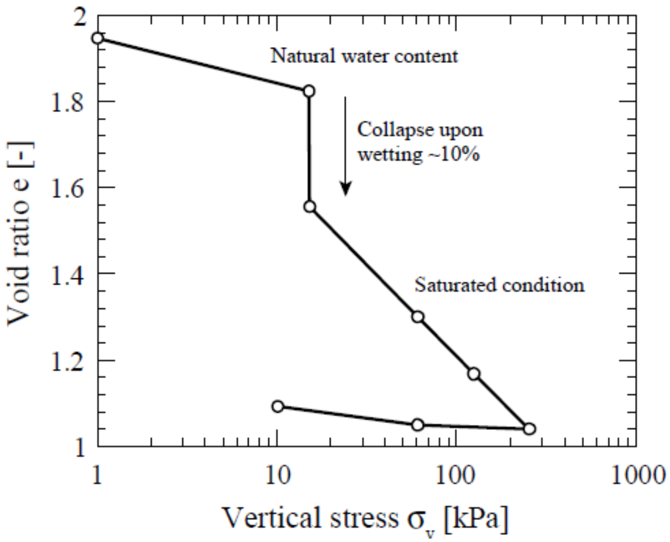
For granular soils and clays with low activity $\chi = S_r$

$$\sigma'_{ij} = \sigma_{net,ij} + Srs\delta_{ij}$$

Volumetric behavior

Collapse upon wetting

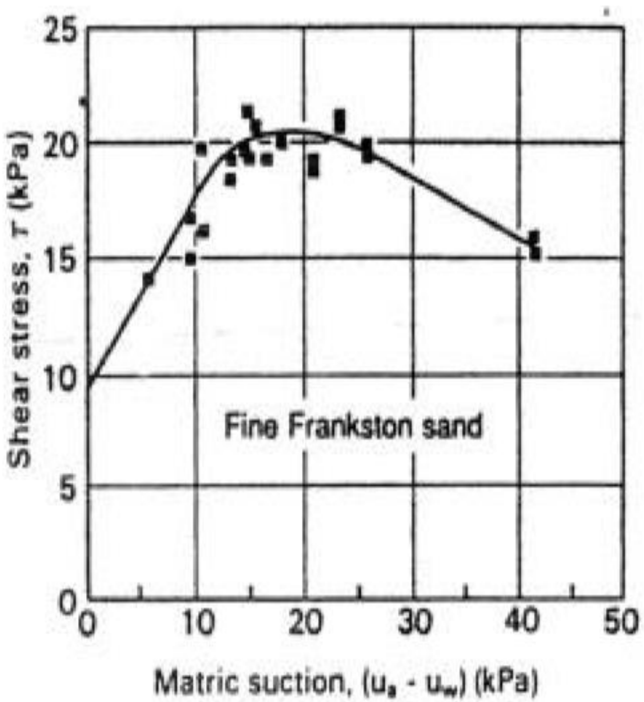


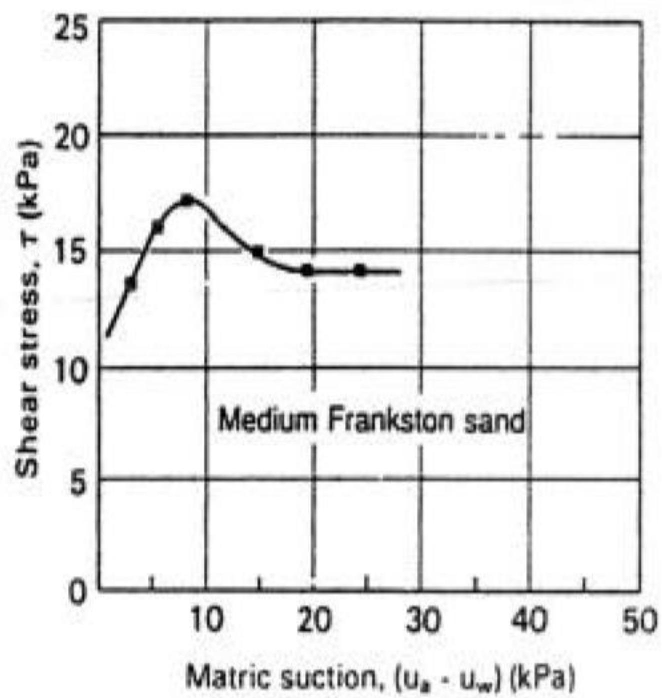


Collapse occurs when total stress is sufficiently high to induce particle slip when the stabilizing effect of menisci disappear

Collapse upon wetting behaviour of a volcanic soil wetted after loading at natural water content (Ferrari et al. 2013)

Shear strength

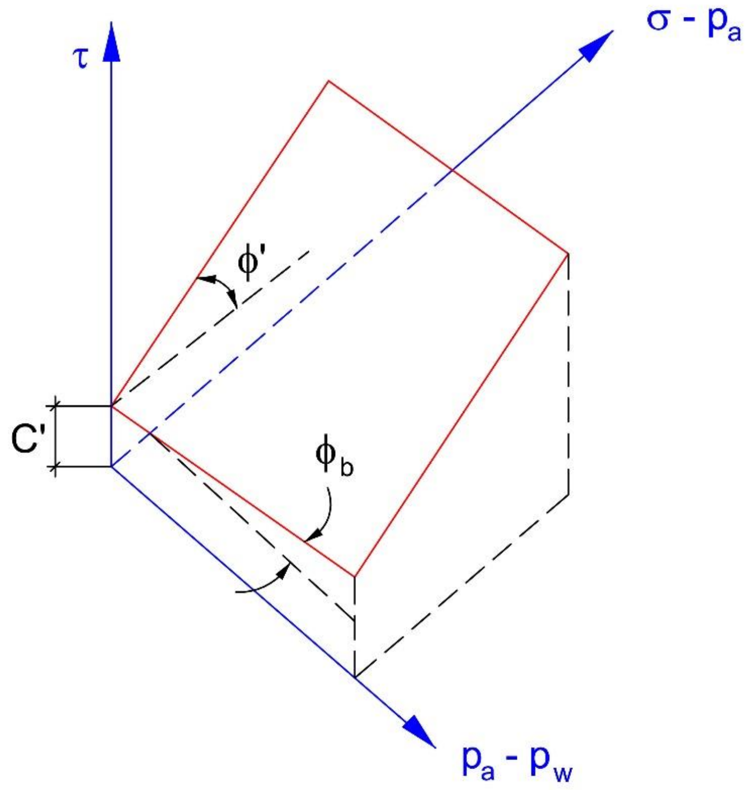




Results of controlled suction direct shear tests on sands (Donald 1956)

Shear strength

Planar shear strength envelope

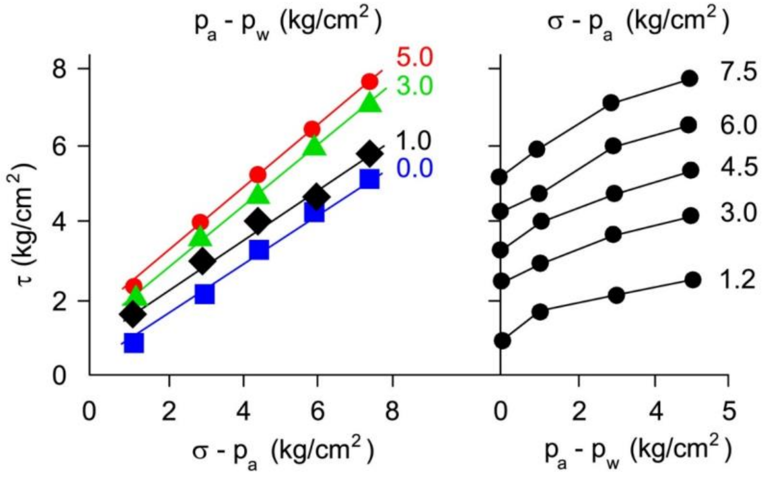


$$\tau_f = c' + (\sigma_n - p_a)\tan\phi' + (u_a - u_w)\tan\phi^b$$

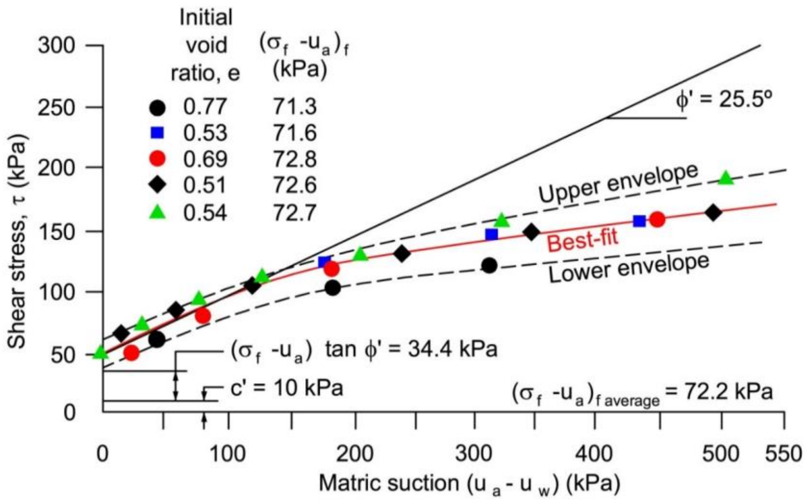
Formulation of shear strength criterion for unsaturated soils by Fredlund et al. (1978)

Shear strength

Curvature of the envelope



Direct shear tests under controlled suction for Madrid clayey sand (“Arena de miga”) (Escario and Saez, 1986)

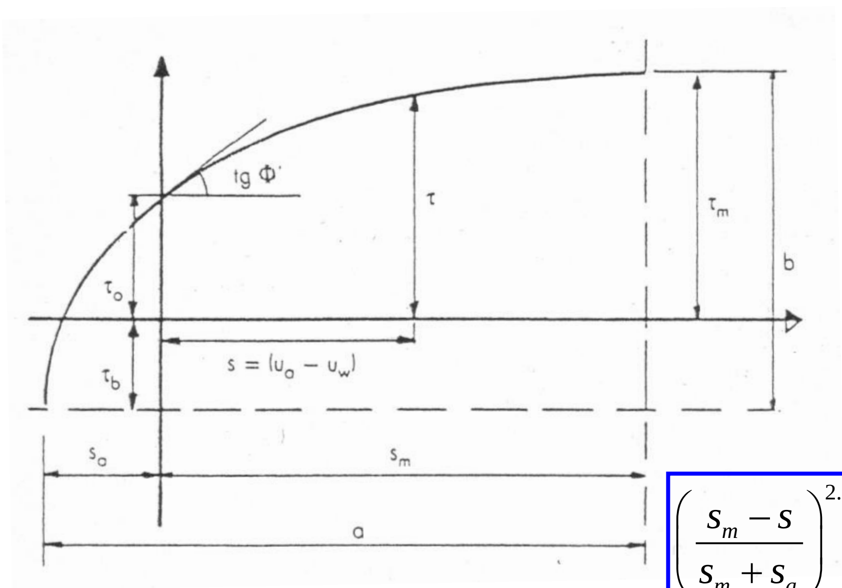


Controlled suction direct shear tests on a glacial till (Gan et al. 1988)

Experimental results show that the increase of strength with suction is not linear (i.e. f_b is not constant)

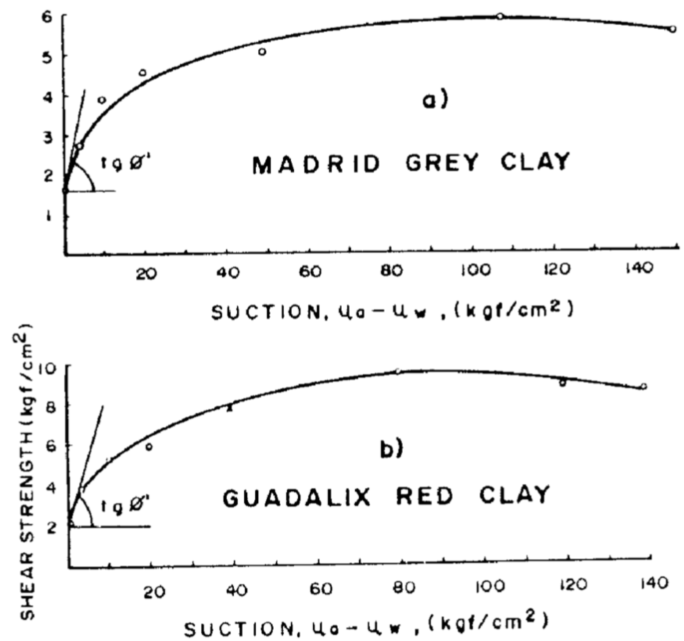
Shear strength

Example of non-linear strength envelope



$$\left(\frac{s_m - s}{s_m + s_a}\right)^{2.5} + \left(\frac{\tau + \tau_b}{\tau_m + \tau_b}\right)^{2.5} = 1$$

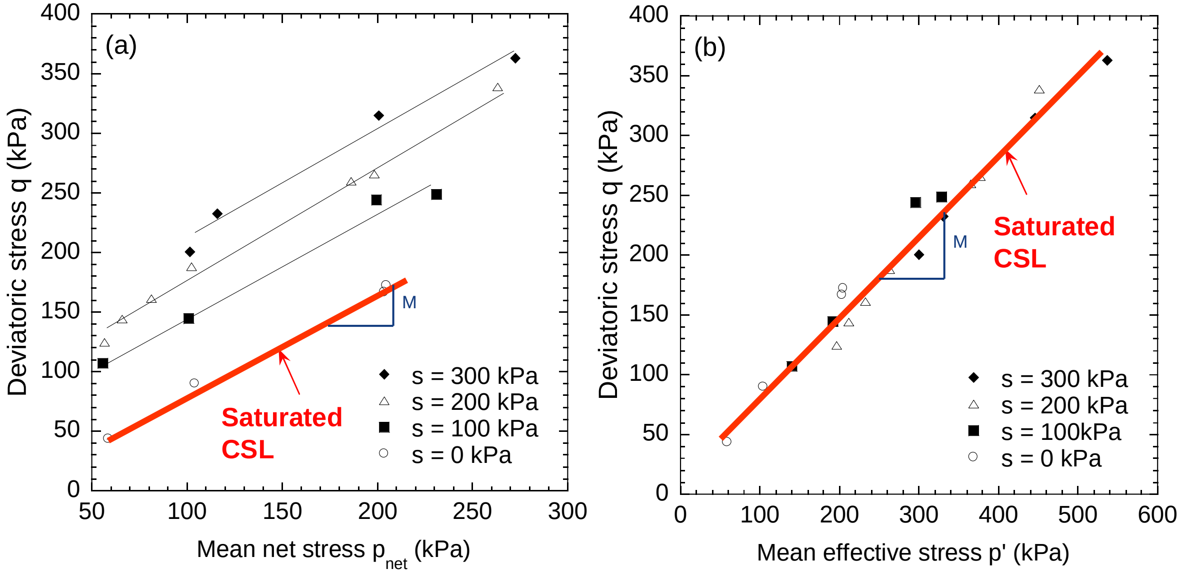
Ellipsoidal shear strength envelope proposed by Escario and Juca (1989)



Data from Escario and Saez (1987)

Shear strength

Critical state behavior



Experimental data from Sivakumar (1993) on kaolin: conventional (q-p_{net}) plane vs effective (q-p') plane

The adoption of a suitable definition of effective stress allows defining a unique shear strength envelope irrespective of the degree of saturation of the soil.
An increase of the mean effective stress (related to an increase in suction or an increase of the mechanical load) results in an increase of the shear strength

Summary

- Unsaturated soils are characterized by two fluids and a solid phase. The negative pore water pressure is usually expressed in terms of suction (i.e. the difference between the pressure of the air and the pressure of the pore water)
- The water retention curve describes how the degree of saturation evolves with suction
- The Darcy's law is still valid for unsaturated soils, the permeability coefficient depends also on the degree of saturation
- The Terzaghi's effective stress is still valid if the pore water pressure is negative and the degree of saturation is equal to 1. Otherwise, a more general expression has to be used
- The volumetric behavior and the shear strength of a soil are affected by variation in suction

Modelling landslides in partially saturated soils

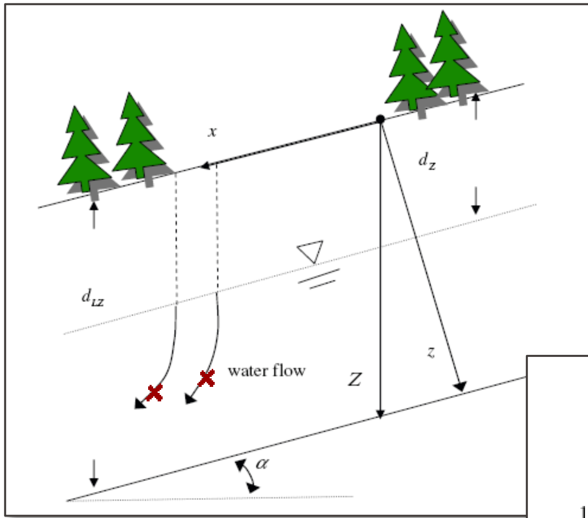
		1D Infiltration Models + Failure Criteria	2D Infiltration Models + LEM	2D Coupled HM + Simple Constitutive Model	2D Coupled HM + Advanced Constitutive Model
H	Initial degree of saturation and suction				
H	Retention properties				
H	Hysteretical features of the retention curves				
H	Permeability evolution with the degree of saturation				
M	Effects of suction and degree of saturation on the constitutive behaviour				
M	Shear strength dependency on suction				
HM	Retention behaviour evolution with void ratio				
HM	Collapse-upon-wetting behaviour				

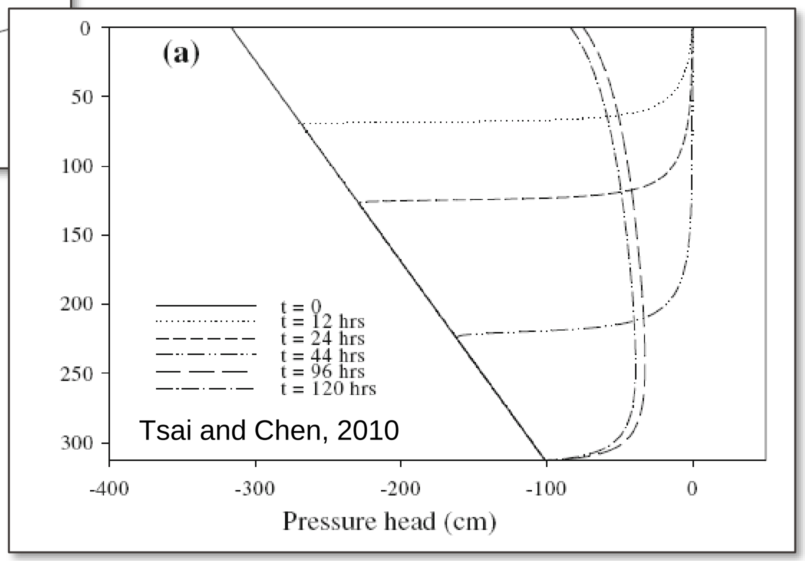
1D infiltration models

1D Richard's Equation

$$\frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = \rho_s A \frac{\partial h}{\partial t}$$

$$A = \frac{\partial \vartheta}{\partial h} \qquad \vartheta = V_w / W_s$$

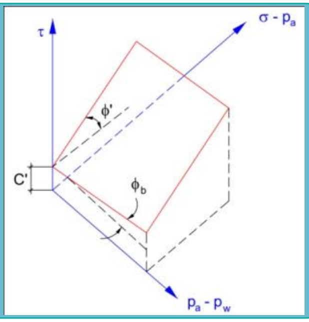


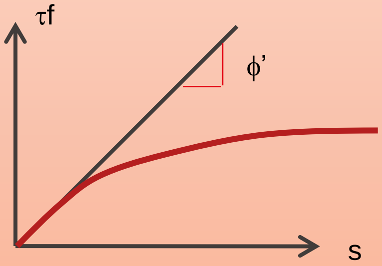


Failure criteria

Fredlund et al. 1978

$$\tau_f = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b$$





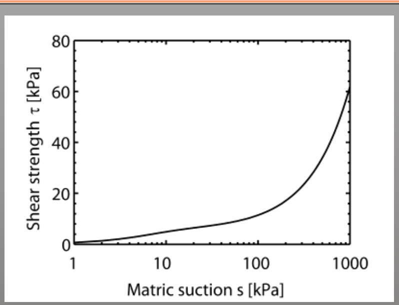
Hyperbolic criteria

$$\tau_f = c' + (\sigma_n - u_a)\tan\phi' + \frac{s}{\cot\phi' + \frac{s}{c^*}}$$

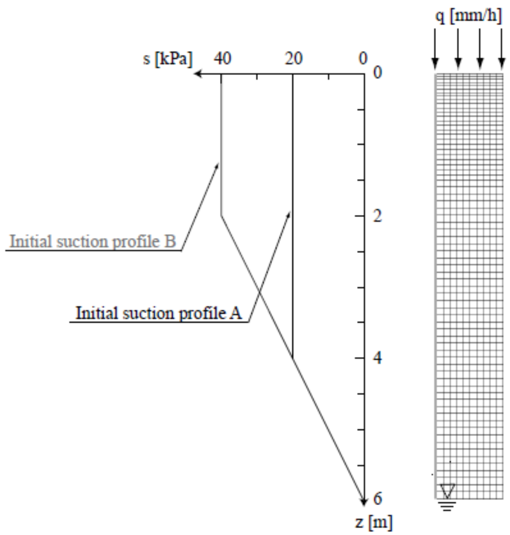
Bishop's effective stress

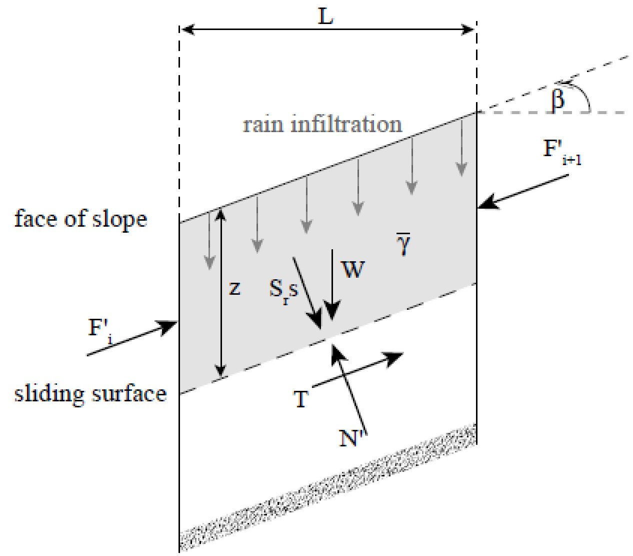
$$\sigma'_n = \sigma_n + S_r s$$

$$\tau_f = c' + (\sigma_{net} + S_r \cdot s)\cdot tg\varphi'$$



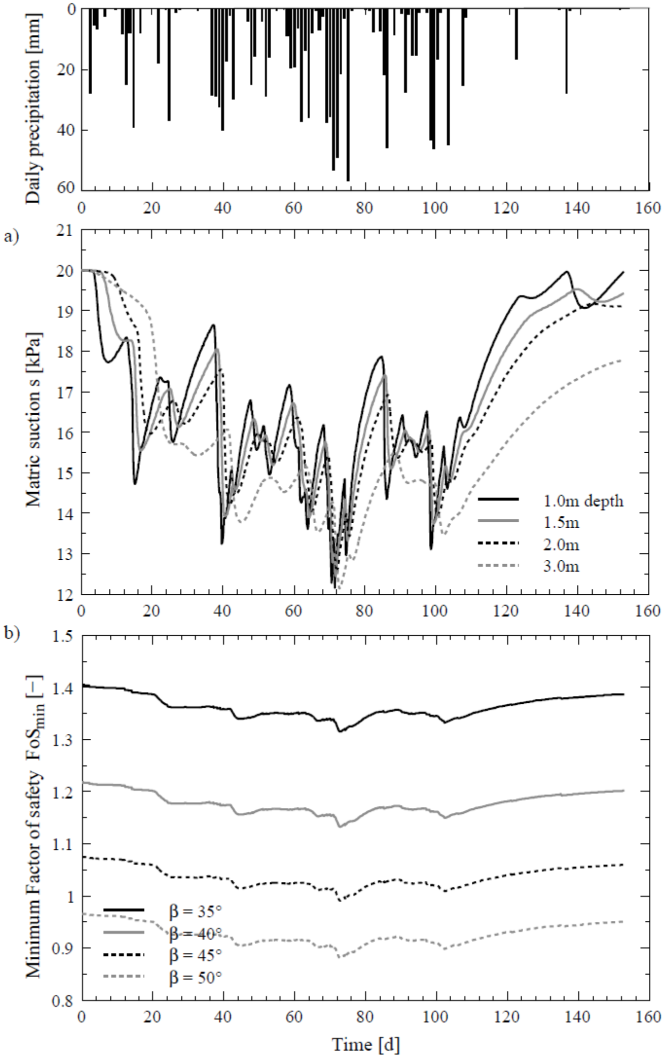
Numerical modelling I: Stability analysis with a soil column model





$$FoS = \frac{c' + \left[\left(\overline{\gamma} z \cos^2\beta - p_a\right) + S_r\left(p_a - p_w\right)\right]\cdot \tan\varphi'}{\overline{\gamma} z \cos\beta \sin\beta} = \frac{c' + \left[\left(\overline{\gamma} z \cos^2\beta\right) + S_r s\right]\cdot \tan\varphi'}{\overline{\gamma} z \cos\beta \sin\beta}$$

(Eichenberger et al. 2013)



Seasonal safety factors

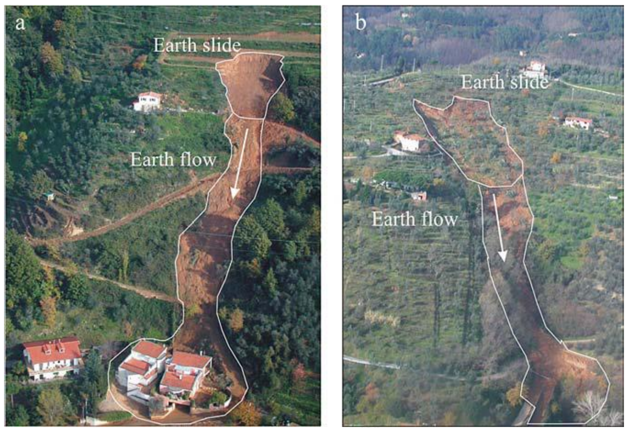
The evolution of matric suction and factor of safety over the wet season show:

1. Worsening conditions in terms of FoS throughout the wet season
2. Importance of antecedent rainfall as predisposition factor for slope failure
3. Shorter dry periods of less than 10 days are not sufficient for a reestablishment of suctions at soil depths below 1m.
4. High intensity rainfall events have a particularly negative effect on slope stability if they occur after 10 days of cumulative rainfall over 70mm.

Modelling landslides in partially saturated soils

		1D Infiltration Models + Failure Criteria	2D Infiltration Models + LEM	2D Coupled HM + Simple Constitutive Model	2D Coupled HM + Advanced Constitutive Model
H	Initial degree of saturation and suction	✓			
H	Retention properties	✓			
H	Hysteretical features of the retention curves	✓ ✗			
H	Permeability evolution with the degree of saturation	✓			
M	Effects of suction and degree of saturation on the constitutive behaviour	✗			
M	Shear strength dependency on suction	✓			
HM	Retention behaviour evolution with void ratio	✗			
HM	Collapse-upon-wetting behaviour	✗			

2D infiltration models + LEM

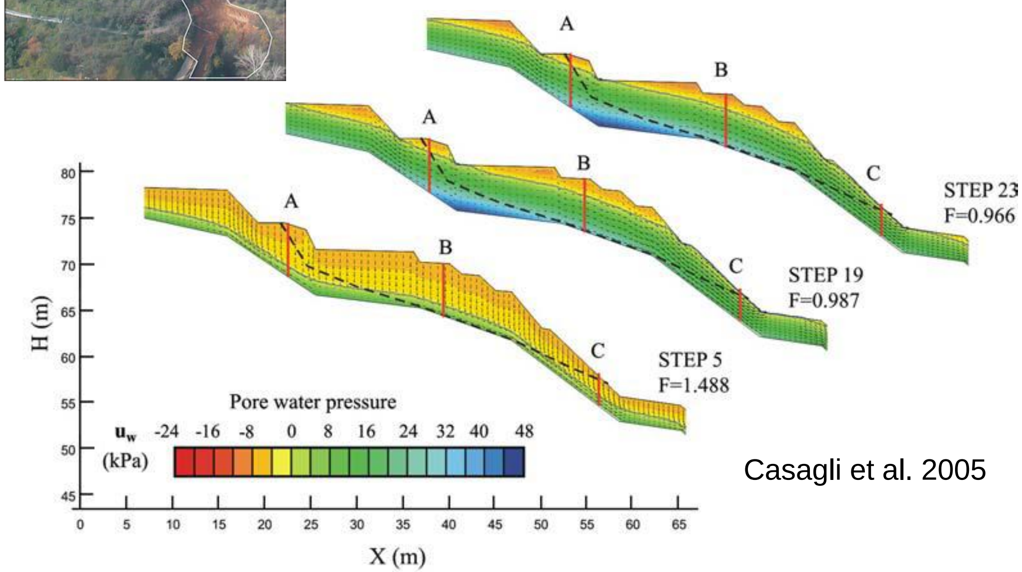


Uzzano landslide (Italy)

2D Richard's Equation

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = \rho_s A \frac{\partial h}{\partial t}$$

$$A = \frac{\partial \vartheta}{\partial h} \qquad \vartheta = V_w / W_s$$



Casagli et al. 2005

EPFL

Modelling landslides in partially saturated soils

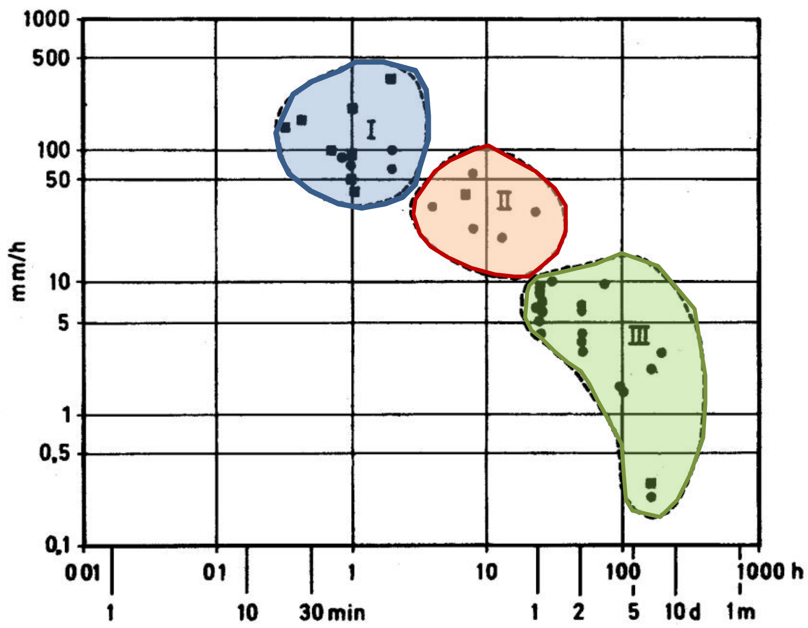
		1D Infiltration Models + Failure Criteria	2D Infiltration Models + LEM	2D Coupled HM + Simple Constitutive Model	2D Coupled HM + Advanced Constitutive Model
H	Initial degree of saturation and suction	✔	✔		
H	Retention properties	✔	✔		
H	Hysteretical features of the retention curves	✔ ✘	✔ ✘		
H	Permeability evolution with the degree of saturation	✔	✔		
M	Effects of suction and degree of saturation on the constitutive behaviour	✘	✘		
M	Shear strength dependency on suction	✔	✔		
HM	Retention behaviour evolution with void ratio	✘	✘		
HM	Collapse-upon-wetting behaviour	✘	✘		

Modelling landslides in partially saturated soils

		1D Infiltration Models + Failure Criteria	2D Infiltration Models + LEM	2D Coupled HM + Simple Constitutive Model	2D Coupled HM + Advanced Constitutive Model
H	Initial degree of saturation and suction	✓	✓	✓	
H	Retention properties	✓	✓	✓	
H	Hysteretical features of the retention curves	✓ ✗	✓ ✗	✓ ✗	
H	Permeability evolution with the degree of saturation	✓	✓	✓	
M	Effects of suction and degree of saturation on the constitutive behaviour	✗	✗	✓	
M	Shear strength dependency on suction	✓	✓	✓	
HM	Retention behaviour evolution with void ratio	✗	✗	✓ ✗	
HM	Collapse-upon-wetting behaviour	✗	✗	✗	

Rainfall induced shallow slips in Alpine regions

- Involved soil masses range between a couple of hundred to thousand cubic meters
- The soil cover in these alpine regions rarely exceeds 2m.
- The reported slips occurred mostly in 1 to 2m depth.



I: mass movements due to rainstorms of high intensity

II: mass movements due to precipitations of several hours' duration with medium intensity

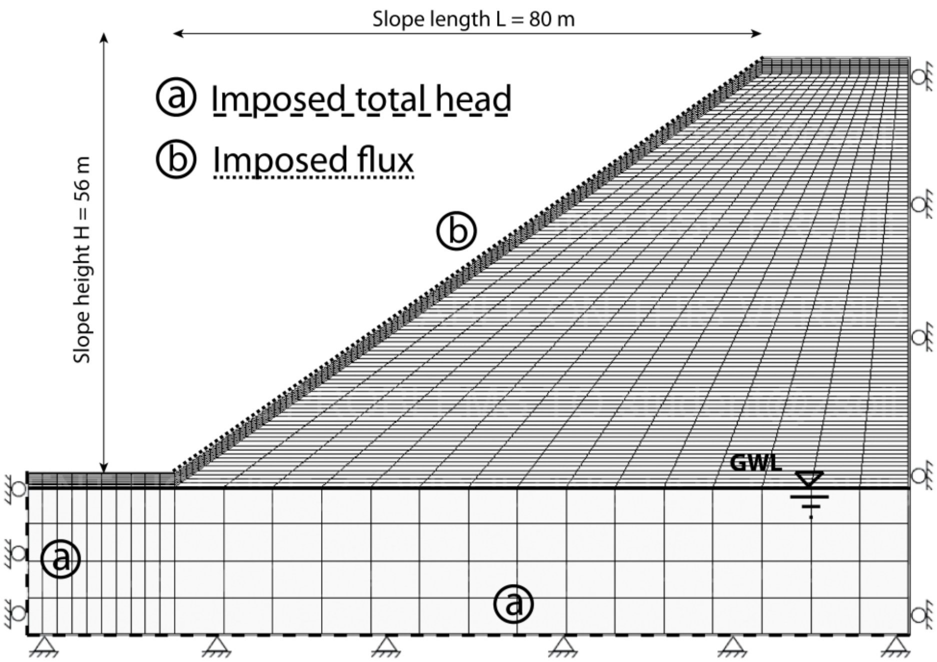
III: mass movements due to continuous heavy rainfall of one to two days duration frequently accompanied by heavy snow melting events

Moser & Hohensinn, 1983

140 events

Geomechanical model: FEM analysis

- 2600 4-noded quadratic elements
- Seepage surfaces are defined along the slope surface
- Biot formulation implemented in Zsoil
- Shear strength reduction for FOS
- Initial state: residual Sr

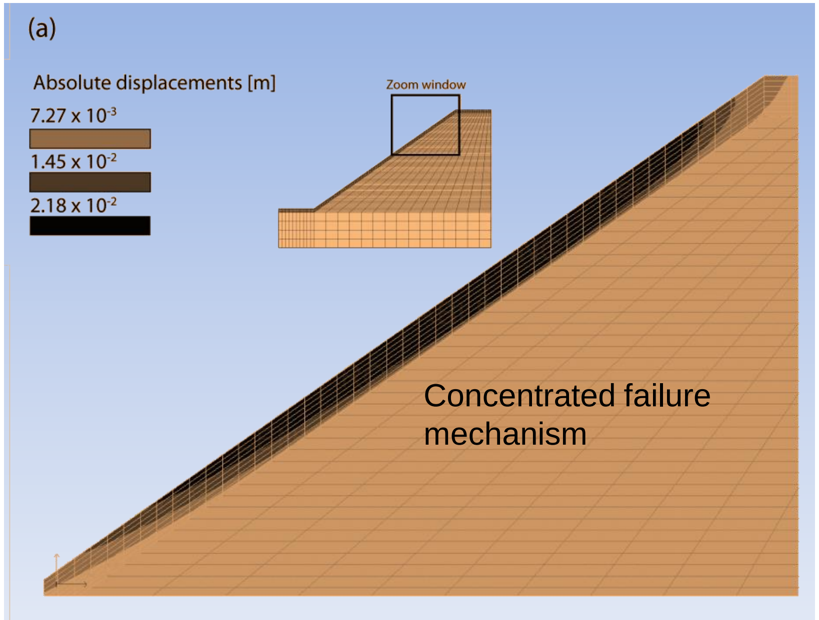


Geomechanical model: material parameters

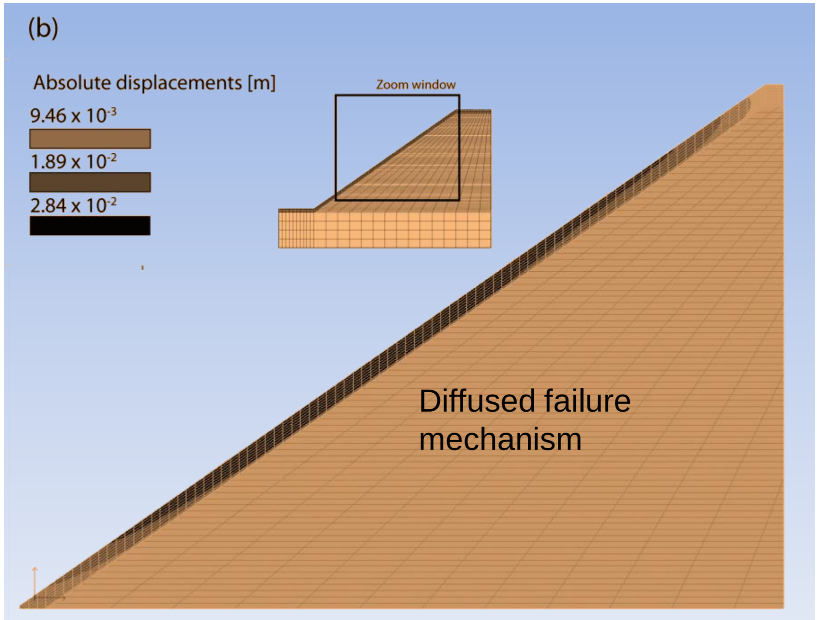
Mechanical parameters		
Elasticity	E : Young's modulus	100 [MPa]
	ν : Poisson's coefficient	0.3 [-]
Plasticity	φ' : shear strength angle	35 [°]
	c' : effective cohesion	0 [kPa]
Weight and density	γ_s : specific soil weight	27 [kN/m^3]
	e_0 : initial void ratio	0.5
Hydraulic parameters		
Retention behaviour	n : Van Genuchten parameter	2 [-]
	m : Van Genuchten parameter	0.5 [-]
	α : Scaling parameter	1.1 [m^{-1}]
	S_{res} : Residual degree of saturation	0.08 [-]
Permeability	$k_{r,sat}$: Saturated hydraulic conductivity	10^{-5} [m/s]

Effects of antecedent conditions

Absolute displacements [m] in the soil cover at the time of failure



Dry condition + major rainfall event of 10 mm/h



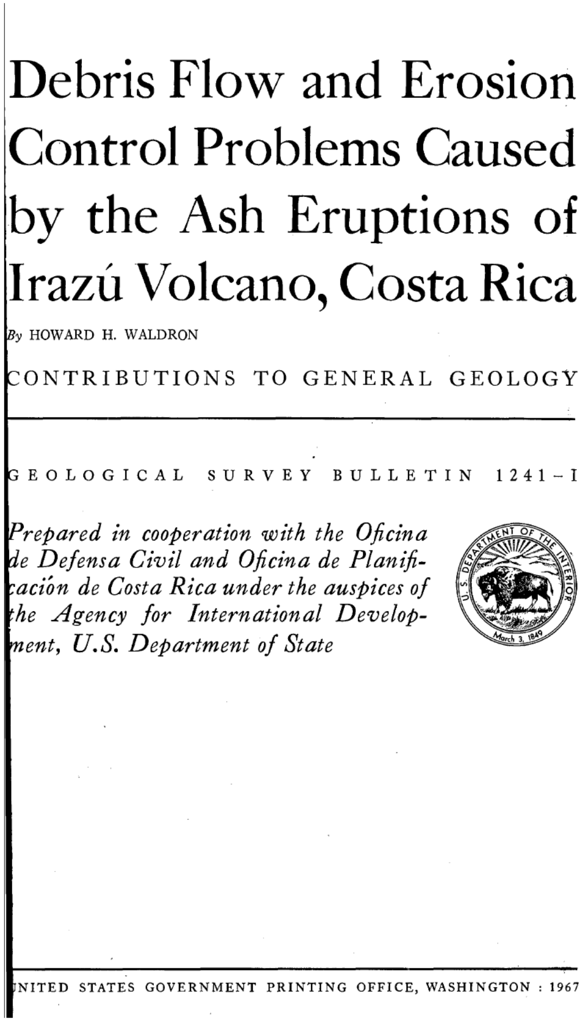
Antecedent water infiltration of 2.5 mm/h over 3 days + major rainfall event of 10 mm/h

Modelling landslides in partially saturated soils

		1D Infiltration Models + Failure Criteria	2D Infiltration Models + LEM	2D Coupled HM + Simple Constitutive Model	2D Coupled HM + Advanced Constitutive Model
H	Initial degree of saturation and suction	✓	✓	✓	✓
H	Retention properties	✓	✓	✓	✓
H	Hysteretical features of the retention curves	✓ ✗	✓ ✗	✓ ✗	✓
H	Permeability evolution with the degree of saturation	✓	✓	✓	✓
M	Effects of suction and degree of saturation on the constitutive behaviour	✗	✗	✓	✓
M	Shear strength dependency on suction	✓	✓	✓	✓
HM	Retention behaviour evolution with void ratio	✗	✗	✓ ✗	✓
HM	Collapse-upon-wetting behaviour	✗	✗	✗	✓

SW hillslope of the Irazu Volcano (Costa Rica)

Scar from a retrogressive landslide on the top hillslope occurred on December 15, 2005 *(photo @Holcim)*


Debris Flow and Erosion Control Problems Caused by the Ash Eruptions of Irazú Volcano, Costa Rica

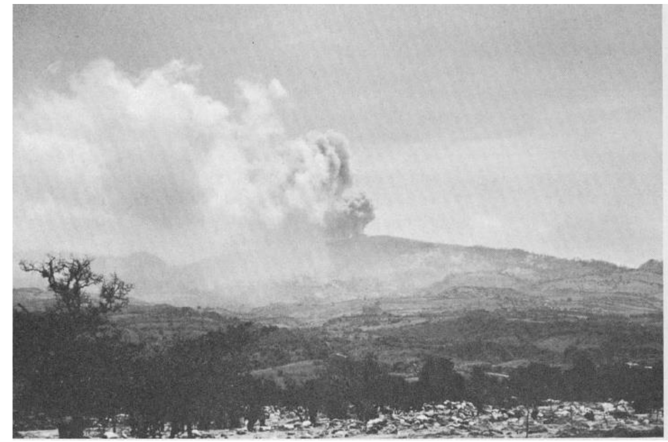
By HOWARD H. WALDRON

CONTRIBUTIONS TO GENERAL GEOLOGY

GEOLOGICAL SURVEY BULLETIN 1241-I

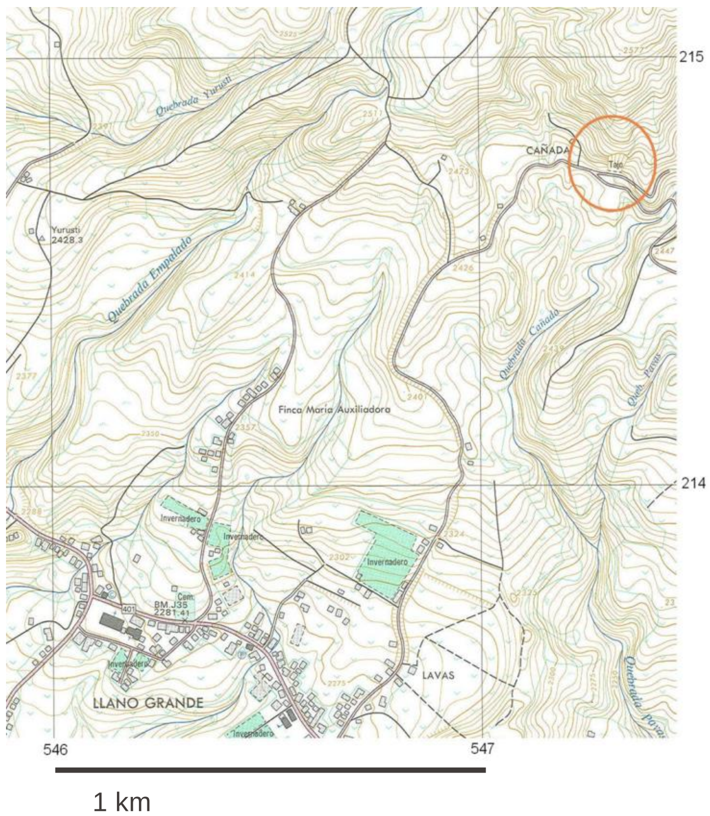
Prepared in cooperation with the Oficina de Defensa Civil and Oficina de Planificación de Costa Rica under the auspices of the Agency for International Development, U.S. Department of State

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1967



- The Irazu Volcano erupted ash almost continuously from March 1963 through February 1965
- Loose ash deposit of up to 30m exist in the surrounding slopes
- Many scars and landslide deposits are reported on the SW flank of the volcano indicating its high landslide susceptibility *(Mora 1985)*

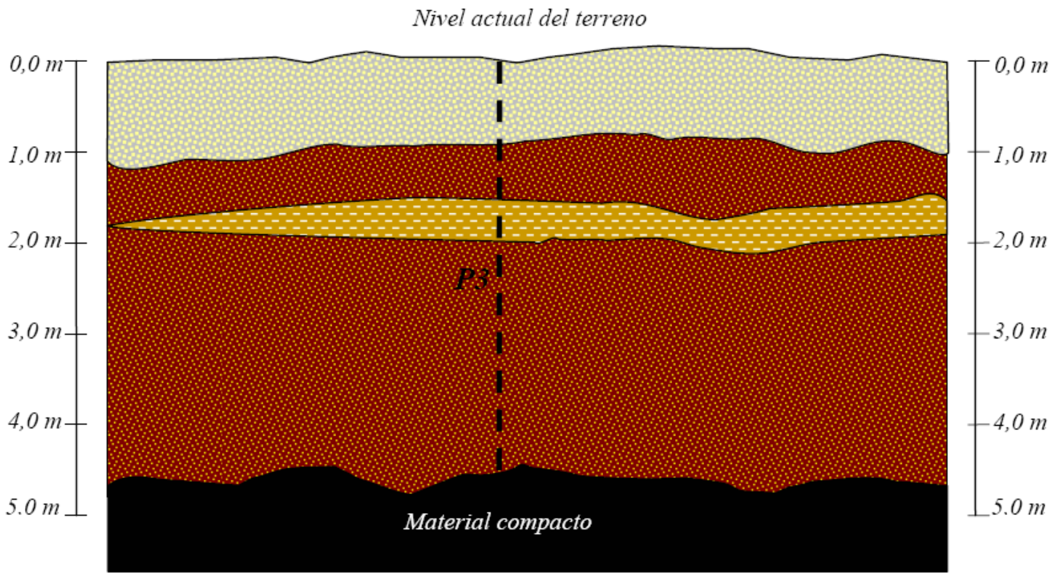
The Llano Grande quarry

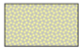


- Ministry of Public Transportation opened the quarry to provide material for the construction of local roads
- In 1992, the site was converted in a pozzolana production quarry
- A large rainfall induced landslide was reported in November 2005 and it was closely followed by a second landslide in December 2005
- In 2010 Holcim Costa Rica decides to establish an earl-warning-system for the mine.

Holcim

The Llano Grande quarry





Descripción: Grava arenosa color café amarillento con partículas café oscuro y beige. Presencia abundante de bloques quebradizos y angulares del orden de mm a dm. No contiene raíces ni materia orgánica.
Plasticidad: nula.
Consistencia: muy firme.
Valores promedio: %w: 21%

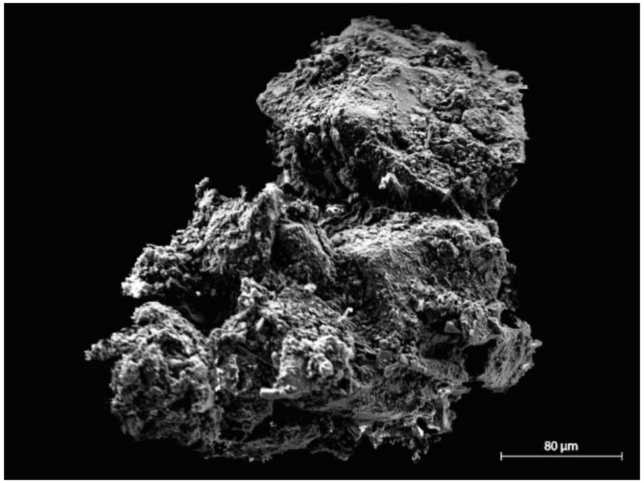
Descripción: Limo café negruzco con partículas beige. Presencia esporádica de bloques quebradizos y angulares del orden de mm a cm. No contiene raíces ni materia orgánica.
Plasticidad: baja a nula.
Consistencia: firme.
Valores promedio: %w: 28; LL: 31; IP: 5
Clasificación SUCS: ML

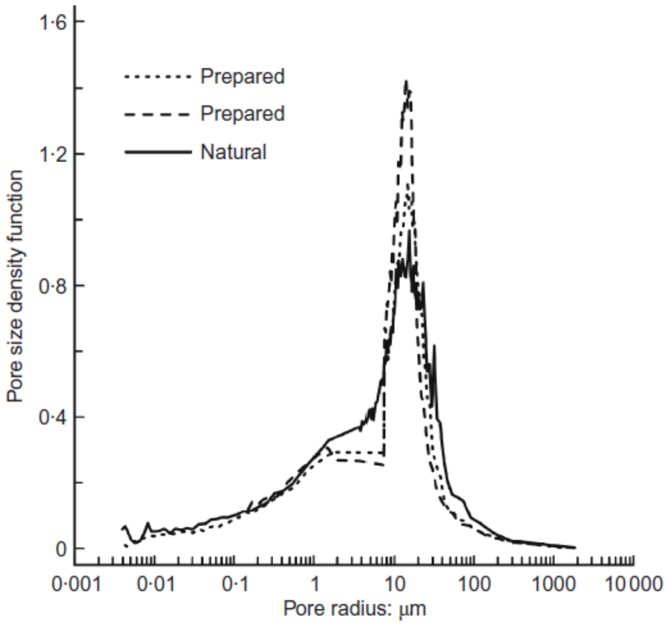
Descripción: Grava arenosa color café amarillento con partículas café oscuro y beige. Presencia abundante de bloques quebradizos y angulares del orden de mm a dm. No contiene raíces ni materia orgánica.
Plasticidad: nula.
Consistencia: firme.
Valores promedio: %w: 23%

Holcim

The volcanic ash from Irazu Volcano

Microstructural characterization
and index properties

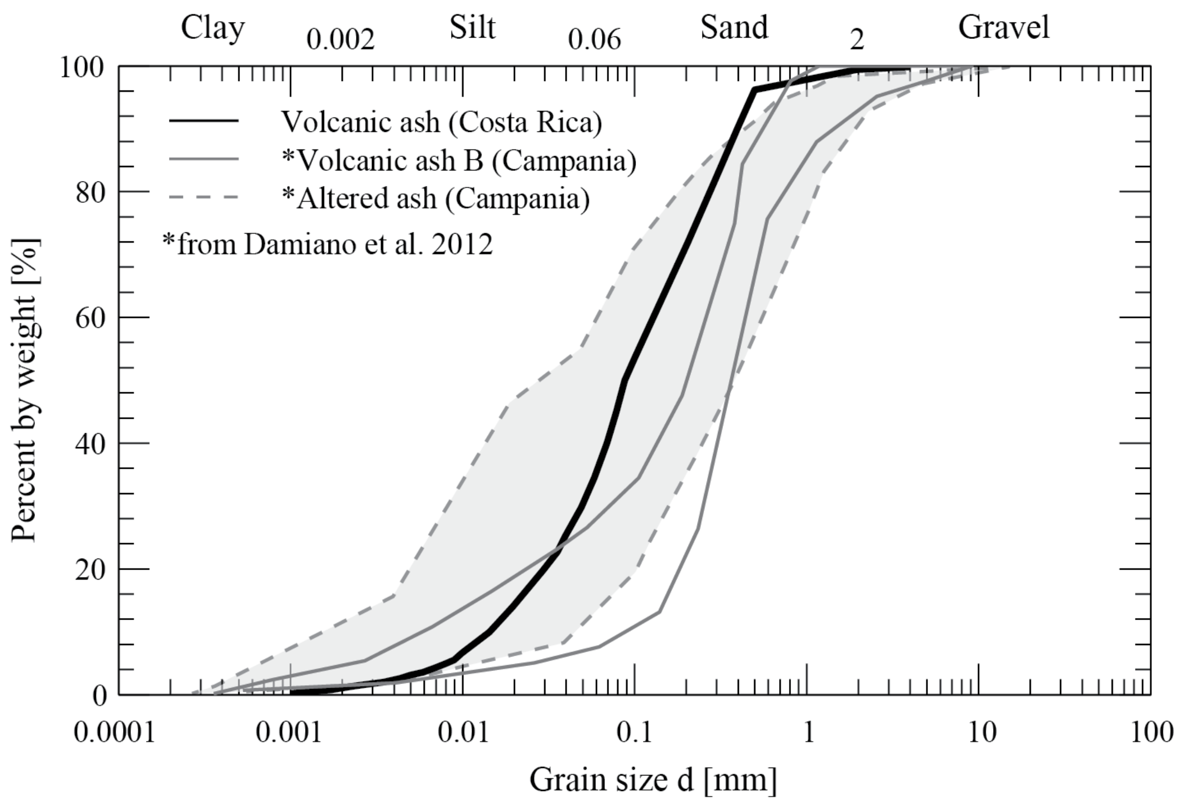




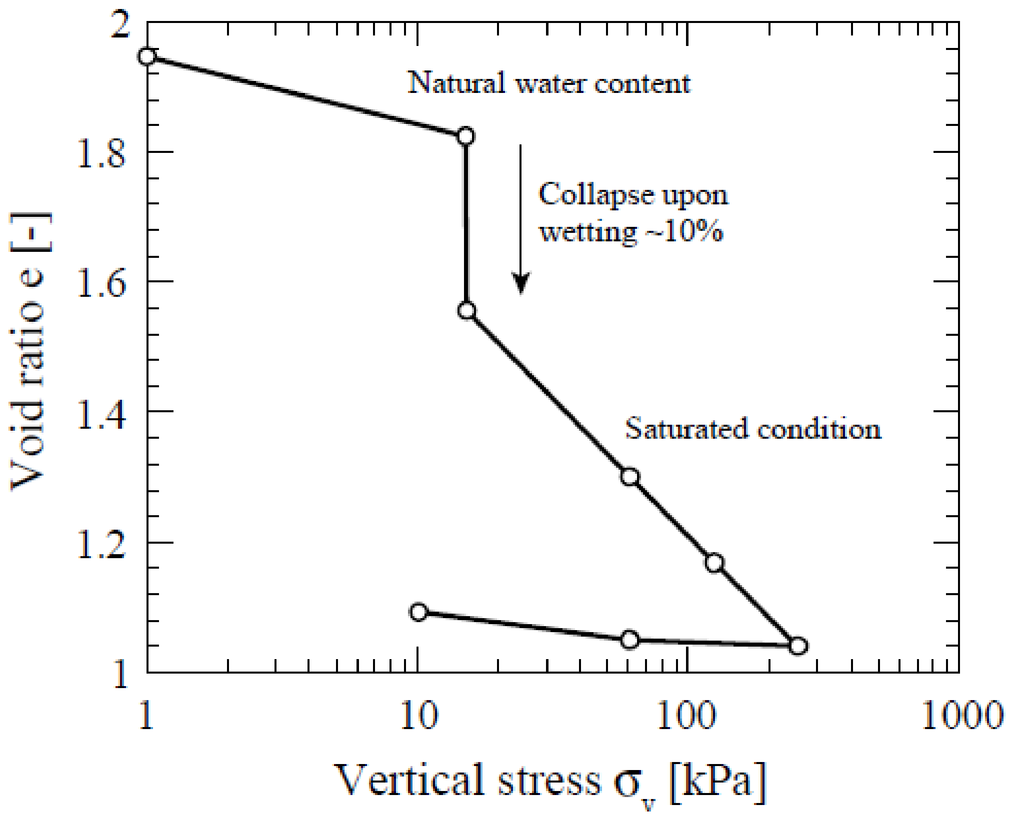
w_n	w_p	w_l	I_p	G_s	γ (kN/m^3)	γ_d (kN/m^3)	e
0.22 - 0.24	0.20	0.26	0.06	2.61	11.6 - 12.5	9.2 - 10.2	~1.6

Ferrari, Eichenberger, Laloui, Géotechnique, 2013

Grain size distribution

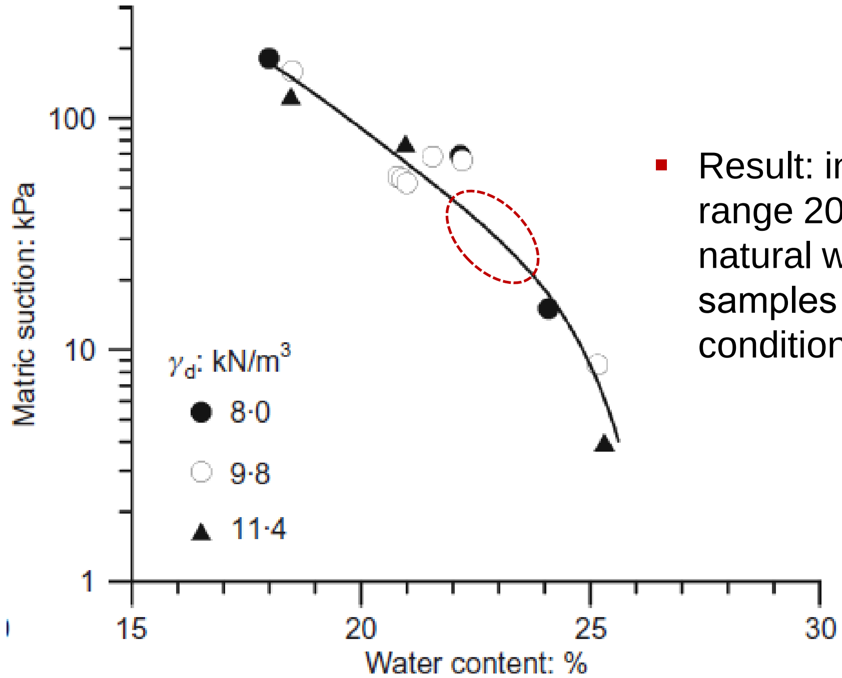


Collapse upon wetting behaviour



Initial suction

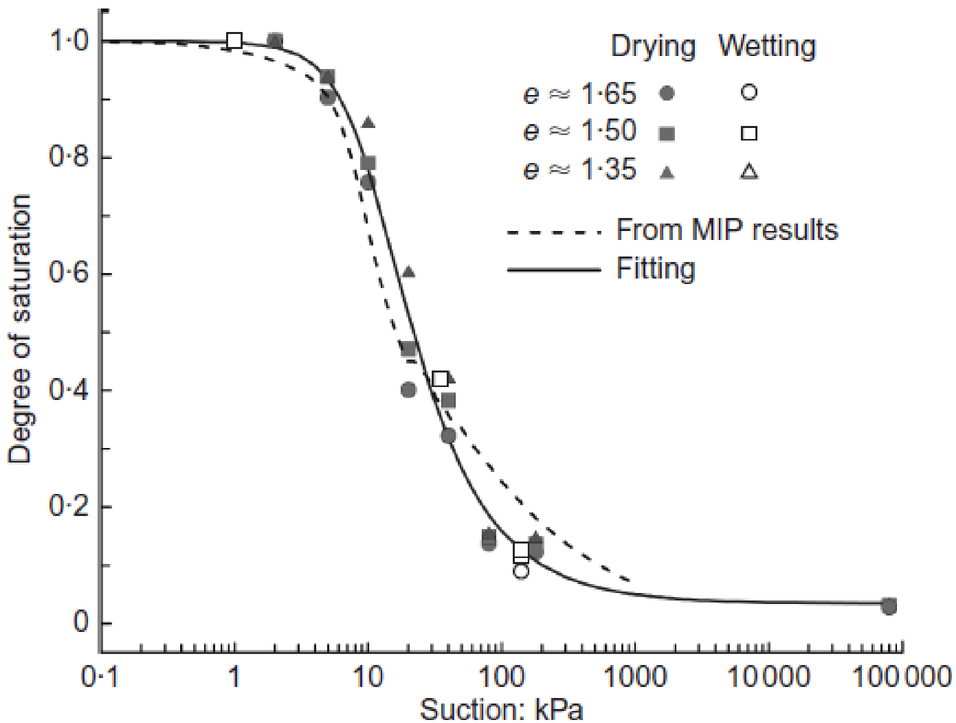
- Measured with contact filter paper and using an ad-hoc calibration for low matric suction values



- Result: initial suction in the range 20–45 kPa for the natural water content of the samples in their 'as delivered' condition

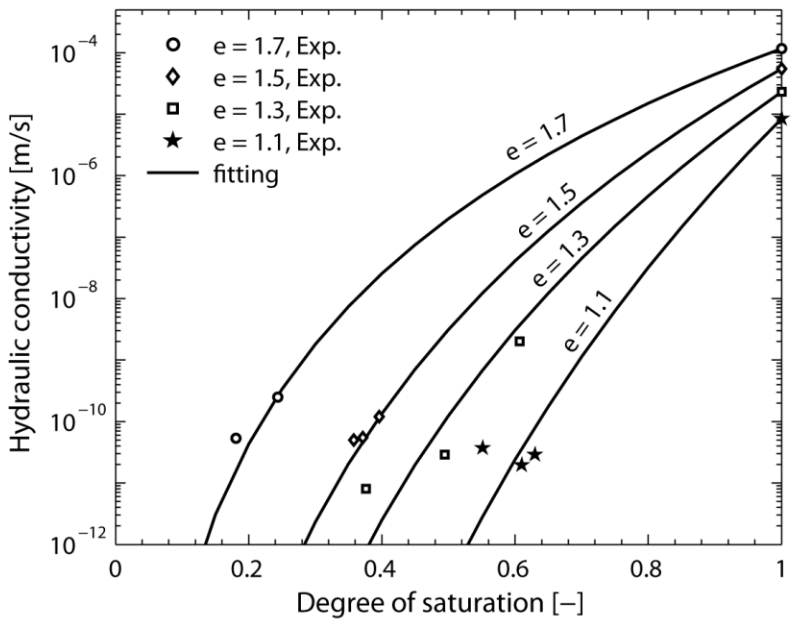
Retention properties

- Combination of pressure plate measurements, controlled-suction oedometric tests and MIP investigations



- No significant dependency on the initial void ratio and no hysteresis between wetting and drying curves

Hydraulic conductivity at variable saturation states



Permeability in variably saturated soil:

$$K(S_r, e_0, \varepsilon_v) = K_{sat}(e_0, \varepsilon_v) \cdot k_{r,w}(S_r, e_0, \varepsilon_v)$$

Saturated permeability:

$$K_{sat} = K_{sat,0}[e_0 - \varepsilon_v(1 + e_0)]^{c_k}$$

Hydraulic conductivity function:

$$k_{r,w} = S_r^{\lambda(e_0, \varepsilon_v)}$$

$$\lambda = c_l[e_0 - \varepsilon_v(1 + e_0)] + c_m$$

S_r : degree of saturation

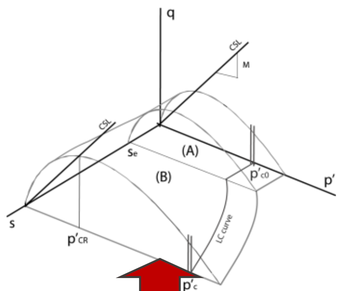
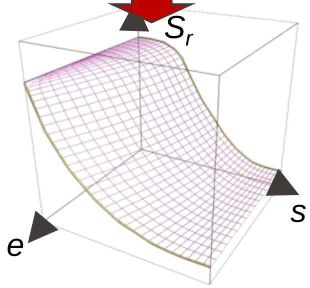
e_0 : reference void ratio

ε_v : volumetric strain

$K_{sat,0}$: saturated permeability at reference void ratio

c_k , c_l , c_m : fitting parameters

Effective stress-strain framework

	Modelled Behaviour	Stress variable	Strain variable	Constitutive relationship
M echanical		$\sigma'_{ij} = \sigma_{net\,ij} + S_r s \delta_{ij}$ Generalised effective stress	ε Skeleton strain	$d\sigma'_{ij} = \mathbf{D}_{ijkl} : d\varepsilon_{kl}$ **D**: material properties
R etention		$s = (p_a - p_w)$ Matric suction	S_r Degree of saturation	$ds = A.dS_r$ *A:* Retention properties

Implicit coupling **Double-way coupling**

The ACMEG-s constitutive modelling framework

Model features:

- Critical state model
- Multimechanism elasto-plasticity
- Bounding surface plasticity
- Suction effect on preconsolidation pressure
- Non-linear elasticity

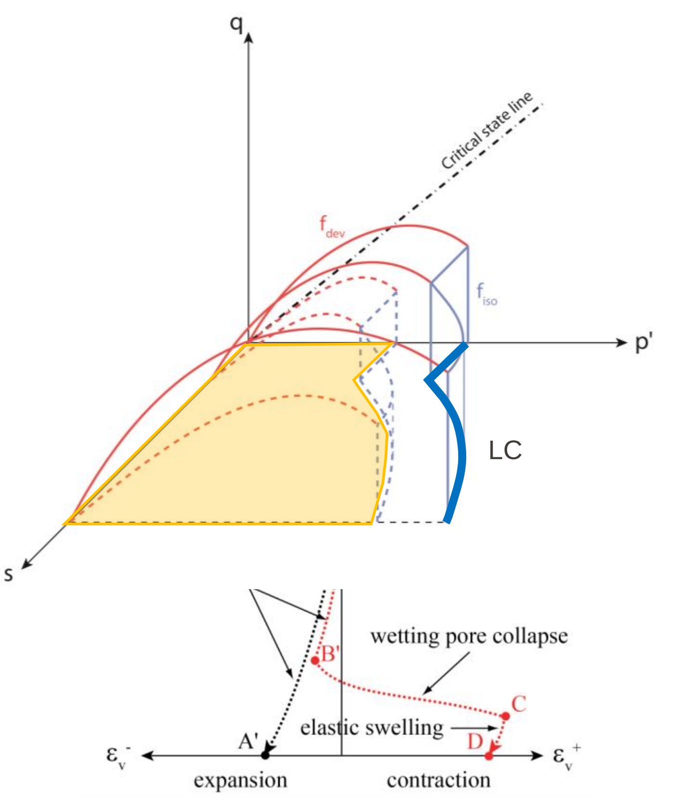
$$d\varepsilon = d\boldsymbol{\varepsilon}^e + d\boldsymbol{\varepsilon}^p$$

$$d\varepsilon_v^e = \frac{dp'}{K} \qquad K = K_{ref}\left(\frac{p'}{p'_{ref}}\right)^{n^e}$$

$$d\varepsilon_d^e = \frac{dq}{3G} \qquad G = G_{ref}\left(\frac{p'}{p'_{ref}}\right)^{n^e}$$

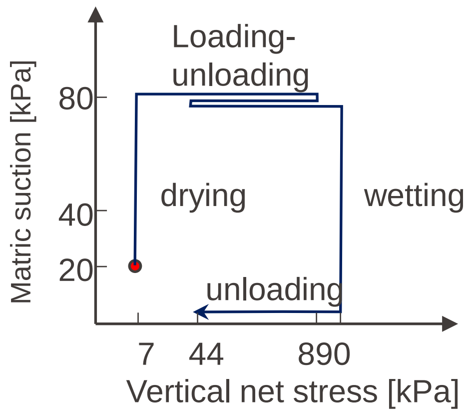
1. Isotropic mechanism with associated flow rule:
$$f_{iso} = p' - p'_c$$
2. Deviatoric mechanism with non-associated flow rule:
$$f_{dev} = Mp'\left(1 - b \cdot log\frac{d \cdot p'}{p'_c}\right)$$

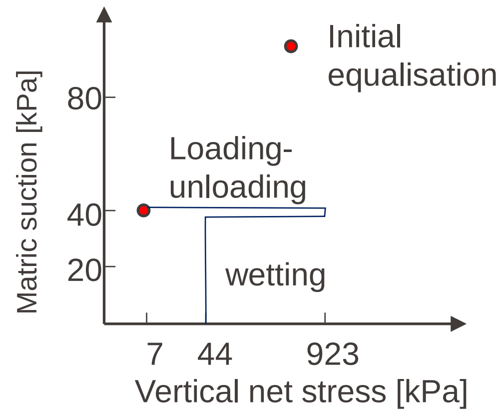
Laloui et al., based on the Hujeux model (1979) and the Loading Collapse concept (Alonso et al. 1990)



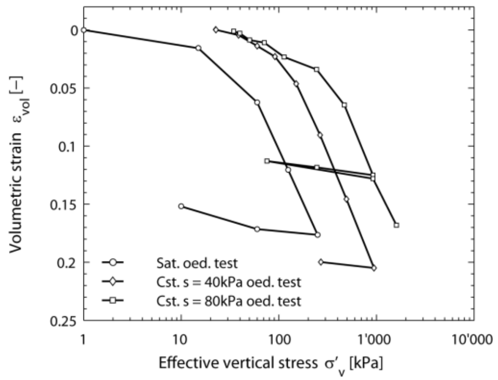
Volumetric response of volcanic ash

Testing program:



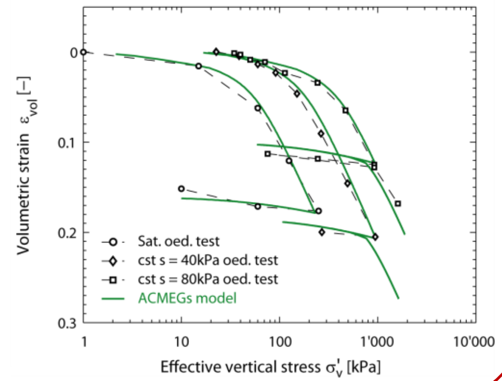


Oedometric response:



Model calibration:

Loading collapse function:

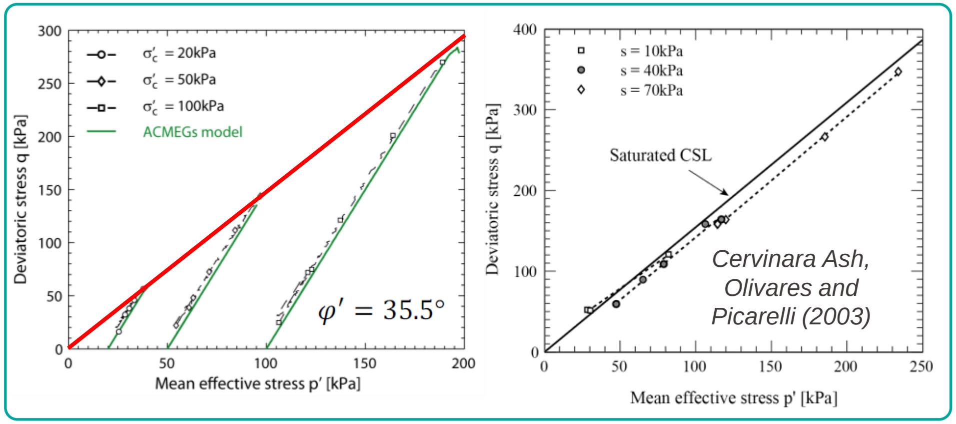


EPFL

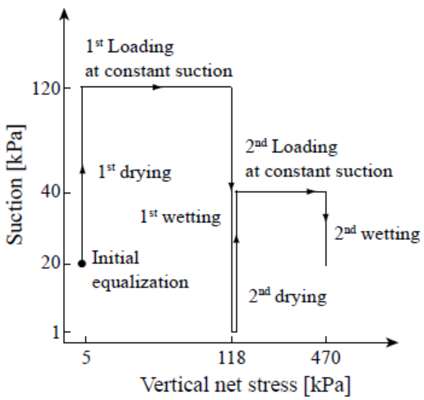
Shear strength of volcanic ash and model validation

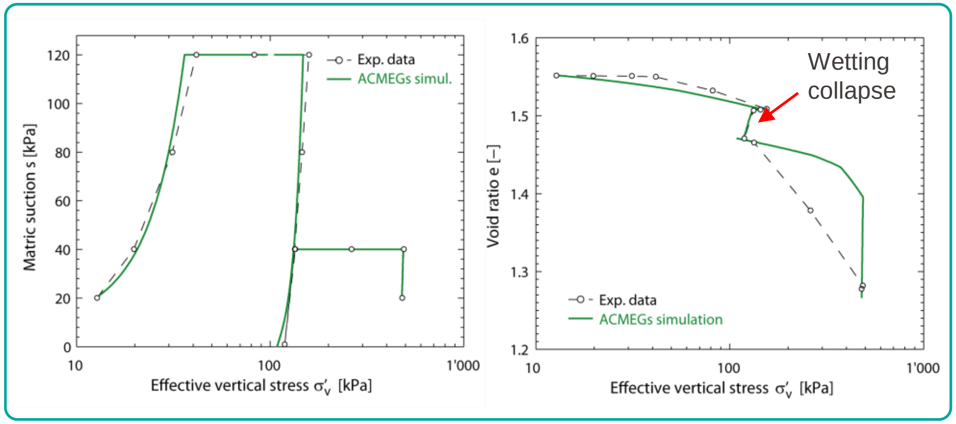
- Shear strength:
  - The saturated failure envelope is also applicable to unsaturated states!

$$q = Mp'$$



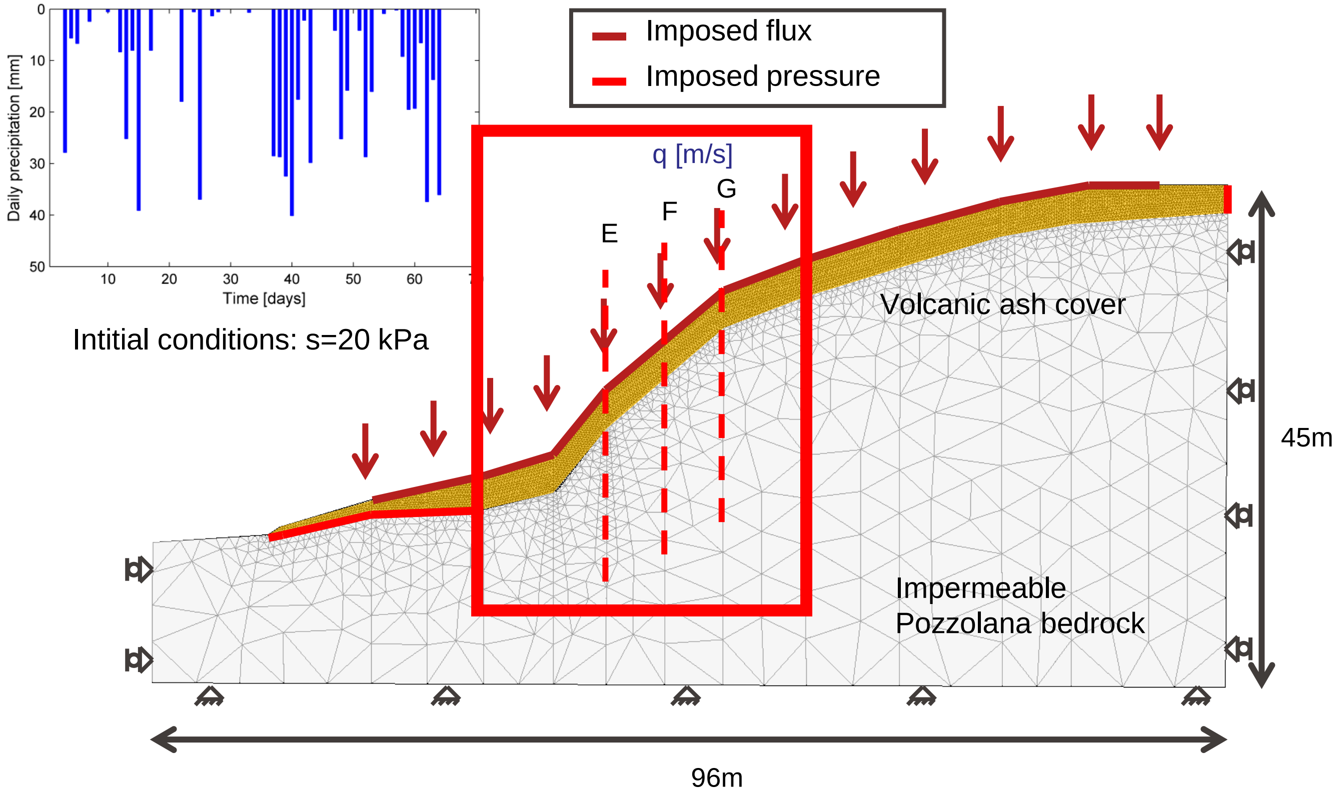
- Model validation:





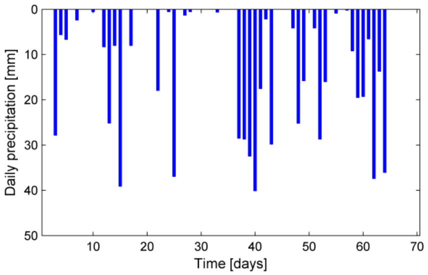
EPFL

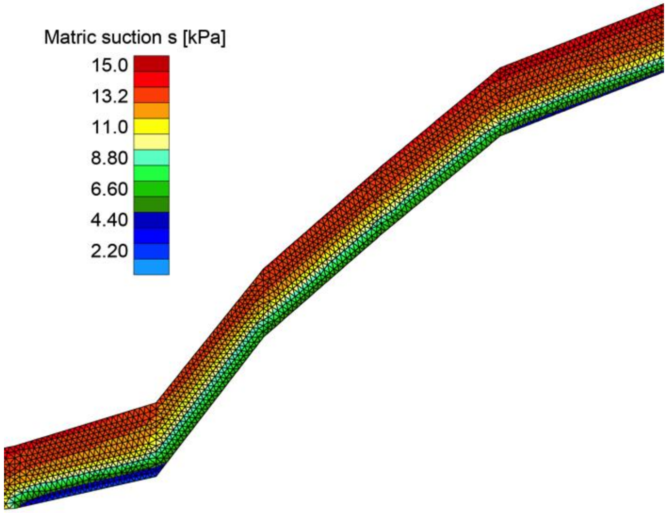
Stress-strain analysis with a two-dimensional slope-scale model

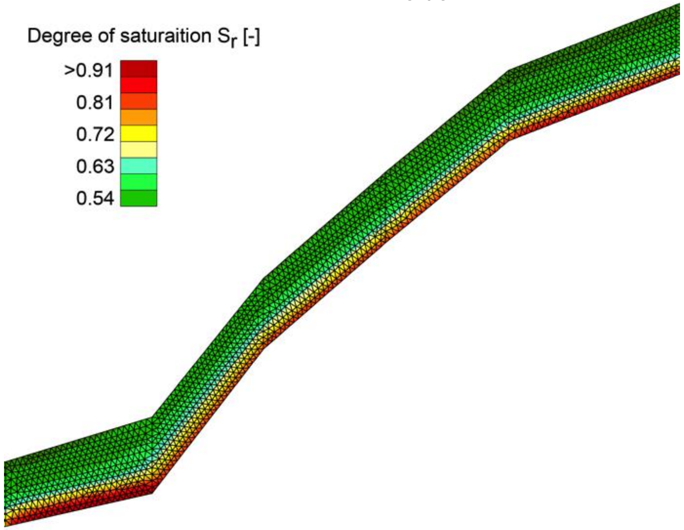


Laloui, Ferrari, Li, Eichenberger, Géotechnique, 2015

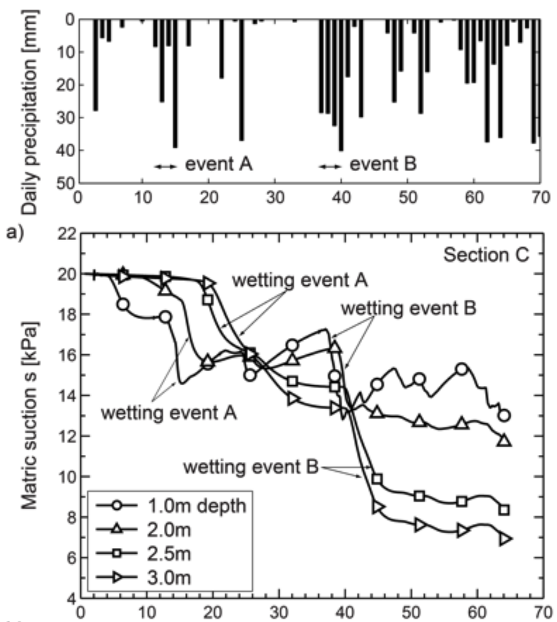
Hydraulic slope response to rain infiltration

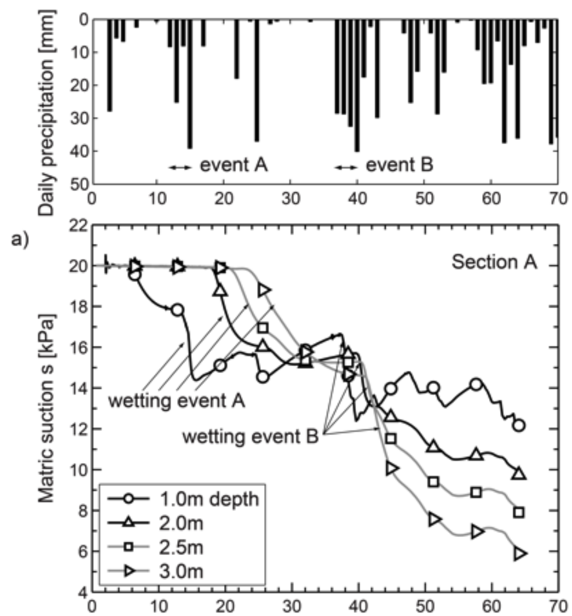


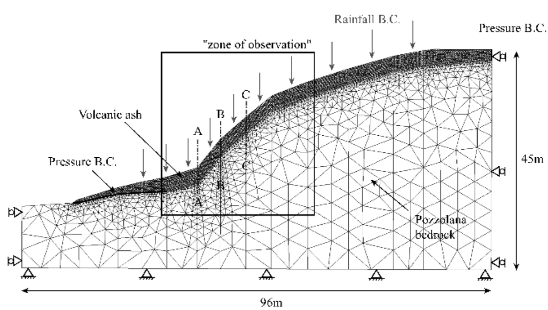




Evolution of matric suction at different depths







Analysis of probable failure mechanism